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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

L. H. DONNELL, *Editor*

T. VON KÁRMÁN, S. TIMOSHENKO, *Editorial Advisers*

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Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

May 1949 Vol. 2, No. 5

Mechanics (Dynamics, Statics, Kinematics)

560. S. A. **Cherkoodinov, "Method of best approximation in the synthesis of mechanisms"** (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekh. Nauk)*, Oct. 1948, no. 10, pp. 1517–1530.

The problem proposed is to approximate a curve (1) $q(x, y) = 0$, where q is a polynomial, by the trajectory (2) $x = x(z)$, $y = y(z)$, (x and y being polynomials in z) of a point of the connecting rod of a four-bar linkage. If the curve (1) is embedded into a family (3) $Q(x, y; t) = 0$, where Q is a polynomial in x, y and $Q(x, y; 0) = q(x, y)$, the Equation (4) $g(z, t) = Q(x(z), y(z); t) = 0$ arises, where g is a polynomial in z and depends also on the dimensions of the linkage. If the latter are so chosen that Equation (4) is satisfied "as best possible," the author regards the problem as solved.

The definition of the best possible approximate compliance with the equation $g(z, t) = 0$, for $z_0 \leqslant z \leqslant z_m$ and $|t| \leqslant L$, claimed to be superior to that of Chebyshev, is the following one: (*a*) The extrema of t shall all have the same absolute value L. (*b*) The number m of the extrema shall be a maximum for all the possible dimensions of the linkage. (*c*) At the end points of the interval (z_0, z_m) the t values are $-L$ and L.

The determination of the "best approximation" proceeds by succesive attempts to make m equal to n, $n - 1$, . . . If $m = n$ is attempted, $z_1, \ldots, z_{m-1}$ must exist satisfying $g(z, L) = (z - z_1)^2(z - z_3)^2 \ldots (z - z_{m-2})^2 (z - z_m)$ and $g(z, -L) = (z - z_0)(z - z_2)^2 \ldots (z - z_{m-1})^2$. Familiar formulas establish then a set of expressions of the values of the coefficients of g at $t = \pm L$ as polynomials in $z_0, \ldots, z_m$. Since these values depend on the linkage parameters, elimination of the z's results in a set of equations for these parameters. If these equations are not compatible, $m = n - 1$ is tried, and so forth.

Examples are given and applications to Watt's linkage and crankshaft curves presented with numerical illustrations.

Continuation of this work in the forthcoming publications of the USSR Academy's seminar in machine and mechanism theory is announced. A. W. Wundheiler, USA

561. P. G. **Bordoni, "Cardanian motions of a double pendulum (Moti alla Cardano di un bipendolo),"** *R. C. Accad. Lincei.*, Sept.-Oct. 1948, ser. 8, vol. 5, sem. 2, pp. 147–154.

The equations of motion of a double pendulum are discussed for two special cases. It is shown that when there is a suitable relation between the geometrical and mechanical parameters, the double pendulum will move as a rigid body, provided its initial orientation and motion is that of a rigid body. Similarly, when another relation between the parameters of the system is satisfied, it may move so that its center of gravity always lies vertically below the point of suspension. For the latter case, exact and approximate evaluations of the integral for the period of oscillation are discussed and a graph is given for the period as a function of the amplitude of either arm of the double pendulum.

The theoretical predictions were investigated experimentally and good agreement between theory and experiment was obtained. L. Landweber, USA

562. L. A. **Waterbury, "Handbook of engineering,"** John Wiley & Sons, New York, 1947. Cloth, 5.1 × 3 in., 386 pp., 126 figs., $2.50.

This is the fourth edition of a pocket-size handbook, amplified by some additions such as the principles of heat transfer. Ahmed D. Kafadar, USA

Theoretical and Experimental Methods

(See also Revs. 562, 568, 573, 585, 668)

563. Callie **Hull, Compiler, "Scientific and technical societies and institutions of the United States and Canada,"** *Nat. Res. Counc. Washington*, bull. no. 115, 1948, 5th ed. Cloth, 10 × 6.75 in., 371 pp., $5.

The new edition of this directory officially sponsored by the National Research Councils of the United States and Canada contains data for 1302 organizations in the United States and for 166 in Canada. The information given in each entry contains: names of officers (as of January 1948), history, purpose, membership and dues, meetings, publications, research funds, medals offered, etc. An index is included which covers subject classifications, changes of name, names of research funds, prizes, medals, etc. and current periodicals published by the societies. L. H. Donnell, USA

564. D. R. **Hartree, M. H. A. Newman, M. V. Wilkes, F. C. Williams, J. H. Wilkinson, and A. D. Booth, "A discussion on computing machines,"** *Proc. roy. Soc. Lond. Ser. A*, Dec. 22, 1948, vol. 195, pp. 265–286.

This is a series of six articles whose titles, in the order of the authors' names given above, are: "A historical survey of digital computing machines," "General principles of the design of all-purpose computing machines," "The design of a practical high-speed computing machine—the EDSAC," "A cathode-ray tube digit store," "The automatic computing engine at the National Physical Laboratory," "Recent computer projects." The discussion is confined to broad principles and intended for the scientist who is a layman in the field of computing machinery. Benjamin Epstein, USA

565. J. Barkley **Rosser, "Theory and application of $\int_0^z e^{-x^2}dx$ and $\int_0^z e^{-p^2y^2}dy \int_0^y e^{-x^2}dx$,"** Mapleton House, Brooklyn, 1948. Cloth, 8.5 × 5.5 in., 192 pp., 8 figs., $8.

This lithoprinted monograph is a report of work done during the war at Allegany Ballistics Laboratory. The single integral $\int_0^z \exp(-x^2)dx$ is essentially the well-known probability integral, which has applications in the theory of probability, statistics, optics, heat transfer and diffusion. Most of these applications have required values of the integral when z is real or purely imaginary. In connection with the mathematical theory of rocket flight there is a need for computing the integral for more general values of z, and also for evaluating the double integral which is specified in the title. This report is concerned with

general methods of calculating these two integrals, particularly when z and p are complex numbers.

The first half of the report discusses known methods of evaluating the single integral and develops theorems to cover more general values of z. The topics treated include computation by means of existing tables, power series expansion, computation by exponentials, use of continued fractions, asymptotic series, and quadratic-equation representation. In the second half, means of computing the double integral are taken up. Principal consideration is given to computation by series; six different types of series are developed and the probable accuracy of computation is discussed. The use of exponentials and continued fractions is extended to the double integral.

As stated in the subtitle of the report, the scope is limited to "Methods of Computation" and no applications are included. However, a number of applications are given in the book *Mathematical Theory of Rocket Flight*, by J. B. Rosser, R. R. Newton, and G. L. Gross, which is also a report of work done at Allegany Ballistics Laboratory. Dana Young, USA

566. Reginald O. Kapp, "The presentation of technical information," Constable & Co., London, 1948. Cloth, 7.5 × 4.8 in., 147 pp., no figs., $1.25.

The title is intended to define more precisely the terms of reference for "what is generally called English," more particularly "functional" (as contrasted with "imaginative") English. Functional English expresses meaning without ambiguity, spares readers unnecessary effort, places every sentence and every word so that it performs its function, presents facts and ideas simply. The only tokens discussed are words, and no use is made of pictures, graphs, or mathematical symbols.

"The aim of a good functional style is to maintain receptivity in the person addressed." Success is less common and more needed than may be supposed. Thought must be conveyed from mind to mind; civilization depends on its being done well.

Carefully built bridges must lead to every deduction. Why are they so often omitted? Through carelessness, or perhaps through fear of making conclusions look too easy. But, alas, sometimes the bridge is omitted from an author's presentation because he has not built it in his own mind.

The book contains discussions rather than lessons, with comments and suggestions, but do's and don'ts only sparingly. In such chapters as "Making it Easy to Understand," "On Meaning What You Say," and "Metaphor," incisive ideas are strikingly expressed. These will be appreciated by technical writers who are making an honest effort to improve their own style more than by beginners who still believe that good style can be taught.

W. P. Roop, USA

Gyroscopics, Governors, Servos

(See Rev. 677)

Vibrations, Balancing

(See also Revs. 577, 678, 680)

567. W. J. Duncan, "Mechanical admittances and their applications to oscillation problems," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2000, Mar. 1946 (issued in 1948), pp. 1–128.

The aim of this monograph is to provide a clear and concise account of mechanical admittances, and to illustrate some of their principal applications to oscillation problems. The exposition is simple and clear, and advanced mathematics has been avoided. No transcendental function beyond the exponential, circular and hyperbolic functions are used. In a few places matrix notation is introduced, but such passages may be omitted without interrupting the main argument.

Many examples have been chosen to illustrate methods and principles, but no attempt has been made to enter deeply into specific technical applications. A special chapter is devoted to approximate methods. Several numerical examples are included. Another chapter is devoted to the experimental determination of admittances.

No doubt this monograph will be of much use to all interested in mechanical vibration problems. Enrico Volterra, USA

568. William T. Thomson, "The Laplace transform solution of beams," *J. acoust. Soc. Amer.*, Jan. 1949, vol. 21, pp. 34–38.

The method of Laplace transforms is applied to the problem of a vibrating beam having concentrated exciting forces, concentrated masses and springs. The author is able to represent the amplitude of the motion by a finite number of trigonometric and hyperbolic terms instead of the usual infinite series solution.

Albert I. Bellin, USA

569. W. L. Hughes, "Propeller blade vibrations," *Trans. N.E. Coast Instn. Engrs. Shipb.*, Mar. 1949, vol. 65, pp. 273–300.

Experiments carried out on a series of model propeller blades are described. The models were vibrated by electromagnetic methods in air and in water. Vibration patterns and specimen diagrams showing the amplitude of vibration at different points of the blade surface are given. Damping of vibrations and typical response curves are discussed. Enrico Volterra, USA

570. L. C. Burrill, "Underwater propeller vibration tests," *Trans. N.E. Coast Instn. Engrs. Shipb.*, Mar. 1949, vol. 65, pp. 301–314.

Experiments on vibrations of marine propellers in water and in air are described. The principal results of the tests can be summarized as follows:

1 The vibration patterns show that the same modes of vibration occur in air and in water and that the corresponding nodal patterns are also similar. But the transverse nodal lines across the blades appeared to be slightly nearer to the shaft axis in the tests in air than in those in water.

2 The effect of water damping on the resonant frequencies is considerable in the lower modes of vibration, and decreases for the higher modes.

3 The ratio of the frequency in water to the frequency in air appears to increase in a linear manner as the number of nodes of each blade increases. Some further tests in air on a flat-plane model of a propeller resembling a marine propeller are also included. In describing these last results some conclusions are drawn regarding blade sensitivity to "singing."

Enrico Volterra, USA

Wave Motion, Impact

(See also Revs. 624, 656, 677, 691)

571. V. V. Sokolovski, "Propagation of elastic-viscous-plastic waves in rods" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, May-June 1948, vol. 12, pp. 261–280. Also *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, May 11, 1948, vol. 60, pp. 775–778 (abbreviated version).

The theory of the propagation of longitudinal stress (σ) waves in rods is developed for materials exhibiting a rate-of-strain effect, with stress-strain relationships of the form:

$$E\frac{d\epsilon}{dt} = \frac{d\sigma}{dt} + \kappa kF\,(|\sigma| - \sigma_S)$$

where F is a monotonically increasing function for positive argument, and zero for negative and zero argument, k is const, and κ is sign σ.

Two uniform stress problems are first considered: a constant strain-rate test, and the effect of a triangular pulse of stress. In each case the resulting stress-strain curve is determined and illustrated for a particular example.

The equations of motion for a rod of varying section are shown to reduce to a pair of quasi-linear hyperbolic differential equations in the dependent variables σ and v (velocity). The characteristics in the x, t-plane are the same as for elastic waves, and the differential relations along them are developed. A shock front of stress discontinuity is also shown to be propagated with the elastic-wave velocity, and the Hugoniot relations of continuity and change of momentum across such a shock are stated.

The variation of a stress discontinuity as it propagates is considered, using the Hugoniot relations across the shock and the relationship imposed along the characteristic coincident with the shock itinerary. The stress variation is thus determined by an ordinary differential equation, the solution being given for particular cases of cylindrical and conical rods. The integrals giving the permanent strain are considered.

Three illustrative problems are treated for the special case in which F is a linear function, the stress, velocity, and strain distributions being determined by numerical integration along the characteristics:

1 A pulse of constant stress on the end of a semi-infinite cylindrical rod. The loading shock wave is propagated at the elastic velocity, the stress magnitude falling as it propagates. Behind the loading wave the stress at any section remains approximately constant until the unloading wave arrives.

2 A triangular pulse of stress, with an instantaneous rise to the maximum value, acting on the end of a semi-infinite conical rod. The stress across the loading shock wave falls off rapidly as it propagates along the conical rod to cross sections of increasing area. For the particular problem considered, plastic flow ceases as soon as the pressure pulse falls below the flow limit.

3 A finite cylindrical rod fixed at one end and free at the other, subjected to a suddenly applied constant stress at the free end. The applied stress has the magnitude 0.75 times the yield stress for the particular problem considered, and plastic flow occurs only after reflection at the fixed end. E. H. Lee, USA

572. F. G. Friedlander, "On the total reflection of plane waves," *Quart. J. Mech. appl. Math.*, Dec. 1948, vol. 1, pp. 376–384.

The well-known equations for reflection and refraction of an oblique plane wave of arbitrary shape at a plane interface between two mediums of different velocities of wave propagation a', a ($a' < a$) break down when the space $y < 0$ containing the incident wave has the smaller velocity of propagation a' and the angle of incidence exceeds the critical angle $\theta_i = \sin^{-1}a/a'$. Snell's law then gives for the sine of the angle of refraction a number greater than 1. The author studies this case and obtains expressions for the reflected wave and for the disturbance in the space $y > 0$.

By way of a definite example the author considers transverse elastic waves; the analysis is equally applicable to other types of elastic waves, such as compressional waves in solids or liquids, or electromagnetic waves.

At $y = 0$ the displacement w is continuous, while the $\partial w/\partial y$ values have a constant ratio μ'/μ. Let w_0, w_1 denote the incident and reflected waves in $y < 0$, and w_2 the disturbance in $y > 0$. Starting with

$$w_0 = f[t - (x \sin \gamma' + y \cos \gamma')/a]$$

for the incident wave, the author shows that

$$w_1 = g\,[t - (x \sin \gamma' - y \cos \gamma')/a]$$

$$w_2 = h\{t - x/c,\ y\,[(1/c^2 - 1/a^2]^{1/2}\} = h(\xi, \eta)$$

where g, h are determined in terms of f, and in particular h is harmonic in ξ, η in the half plane $\eta > 0$, and satisfies the boundary condition

$$\left(\frac{\partial h}{\partial \xi} - \lambda \frac{\partial h}{\partial \eta}\right) = 2f'(\xi)$$

along $\eta = 0$, where λ is a proper constant. This is solved in the form

$$h(\xi, 0) = \frac{2}{1 + \lambda^2}\,[f(\xi) + \lambda' f_c(\xi)]$$

where $f(\xi, \eta)$ is a harmonic function such that $f(\xi, 0) = f(\xi)$, $f_c(\xi, \eta)$ is the conjugate harmonic to $f(\xi, \eta)$, $f_c(\xi) = f_c(\xi, 0)$, so that f_c is the Cauchy principal value of

$$(1/\pi) \quad {}_{-\infty}\int^{\infty} [f(\alpha)/(\alpha - \xi)]d\alpha.$$

It is shown by applying the above that even when the incident wave shape f has a well-defined front, the transmitted and reflected disturbances fail to possess a definite front.

H. Poritsky, USA

573. J. Hadamard, "On the abnormal case of the Cauchy problem for the wave equation (Sur le cas anormal du problème de Cauchy pour l'équation des ondes)," *Courant Anniv. Vol.*, Interscience Publishers, New York, 1948, pp. 161–165.

The author considers for the equation $u_{xx} + u_{yy} + u_{zz} = u_{tt}$ (or $u_{xx} + u_{yy} = u_{tt}$) the Cauchy problem when the initial manifold is timelike (that is, its normal makes an angle less than 45 deg with the t-axis). He concentrates on the three-dimensional case when, for $x = 0$, $u = U$ is given and $u_x = 0$.

A solution W can be obtained by taking the hemispherical (or hemicyclic) mean value of U. To answer the question whether W is arbitrary he states that U can be determined from W by means of the operators C_y and C_z defined by:

$$C_\alpha W = \frac{\partial}{\partial \alpha}\int_0^t t^2 W(y, z, t)dt + \alpha W, \text{ where } \alpha \text{ is } y \text{ or } z.$$

W is determined inside the characteristic double cone (whose axis is parallel to the t-axis) by its values in the neighborhood of its axis. Therefore U is not arbitrary—at variance with the normal Cauchy problem.

In general, if W is given in a domain D_0, it can be continued analytically into the Lorenz-invariant domain formed by the spacelike parts of the boundary of D_0 and bounded by the two developable characteristic surfaces circumscribed around D_0.

A. W. Wundheiler, USA

Elasticity Theory

(See also Revs. 582, 589, 602, 611, 626)

574. M. Reiner, "Elasticity beyond the elastic limit," *Amer. J. Math.*, Apr. 1948, vol. 70, pp. 433–446.

The author begins by distinguishing between "deformation" and "strain," denoting by strain that part of a deformation which is recovered when all external forces are removed; however, his

work applied equally to those cases when the so-defined strain and deformation are identical. On the assumption that there is a one-to-one relation between the stress tensor and the strain tensor it is shown that this relation can be expressed in the forms

$$T_r^s = (F_{01}I_e + F_{02}II_e + F_{03}III_e)\delta_r^s + F_1 e_r^s + F_2 e_\alpha^s e_r^\alpha$$

$$e_r^s = (\Phi_{01}I_T + \Phi_{02}II_T + \Phi_{03}III_T)\delta_r^s + \Phi_1 T_r^s + \Phi_2 T_\alpha^s T_r^\alpha,$$

where I_e, II_e, III_e, are invariants of the strain tensor, I_T, II_T, III_T, are invariants of the stress tensor, the F's are functions of the strain invariants and the Φ's are functions of the stress invariants.

The above relations are independent of any particular measure of strain, and the author chooses the Hencky (or logarithmic) measure in expanding his theory. By considering certain special types of stress fields the F's are given meanings as generalized moduli of elasticity and the Φ's as generalized coefficients of elasticity. The reviewer finds some difficulty in appreciating the worth of these moduli and coefficients since neither F_{01}, F_{02} and F_{03} nor Φ_{01}, Φ_{02} and Φ_{03} appear to be uniquely defined.

D. N. De G. Allen, England

575. W. H. Wittrick and W. Howard, "Relaxation methods applied to two problems of two-dimensional stress distribution involving mixed boundary conditions," *Austral. J. sci. Res. Ser. A*, June 1948, vol. 1, no. 2, pp. 135–160.

Two new examples are solved of stress determination in flat plates subjected to "mixed" boundary conditions by a straightforward application of the relaxation method which was previously described by Fox [*Proc. roy. Soc. Lond. Ser. A*, 1947, vol. 189]. D. N. De G. Allen, England

576. Guthlac Wilson, "A relaxation method for the solution of problems concerning axially symmetrical distributions of load in an elastic medium," *J. Instn. civ. Engrs.*, Apr. 1948, vol. 30, pp. 149–166.

The problem of this paper is to determine the displacements and stresses in an isotropic perfectly elastic half-space body, which are produced by a rigid disk loaded axisymmetrically and located either at the surface or at a chosen depth below the surface.

The author chooses as a method of analysis the (approximate) transformation of the differential equations, stated in terms of the radial and axial displacements, into difference equations. A square grid in the radial planes through the axis of symmetry is used, and it is implied that this defines a sector-ring grid in the planes perpendicular to the axis of symmetry. Near the axis of symmetry ($r = 0$) the author changes the equations, "taking limits" without explaining his limit procedure.

The set of difference equations thus obtained is then successively solved for the displacements, and from them for the stresses. The numerical results are compared with good agreement to the results of an exact integration of the differential equations.

Such methods do not seem to use the artifice of fixing all points of the grid except one relaxed point or group of points, and then of relaxing other points or point groups, after fixing the first point or group. It therefore does not seem clear why such a method is called one of relaxation. Hans J. Reissner, USA

577. Eric Reissner, "Note on the method of complementary energy," *J. Math. Phys.*, July 1948, vol. 27, pp. 159–160.

The author considers the forced oscillations of an elastic body (in general anisotropic) under the influence of prescribed surface stresses and body forces, both of these having a simple harmonic time-dependence (time-factor cos ωt). He considers a class C of states in which the stress and the displacement have this time factor and are otherwise arbitrary except that (a) the equations of motion are satisfied, and (b) the boundary conditions are satisfied. In general, the equations of compatibility are not satisfied, so that stress and displacement are independent.

A strain-energy function of position W is defined by the usual quadratic expression in terms of stress, but with the time factors omitted, and a function of position K is defined by

$$K = {}^1/_2\rho\omega^2(u^2 + v^2 + w^2),$$

where ρ is density and (u, v, w) the displacement with the time factor omitted. The integral $\int(W - K)dV$, taken throughout the body, is considered for the class of states C, and it is proved that this integral has a stationary value when the equations of compatibility are satisfied by the stress, and the displacement is that which corresponds to the stress through the expressions for strain and the stress-strain relations, that is when the state of the body is the actual state of vibration.

Courtesy of Mathematical Reviews J. L. Synge, Ireland

578. G. B. Pacella, "On a property of the mechanics of continuous bodies and a geometric consequence of Hooke's law (Su una proprieta della meccanica dei corpi continui e una deduzione geometrica della legge di Hooke)," *R. C. Accad. Lincei*, July-Aug. 1948, ser. 8, vol. 5, sem. 2, pp. 31–37.

At a given point of a stressed continuous medium, plane elements carrying the same normal stress envelop a cone, which intersects the unit sphere centered at the point in a quartic curve. These curves, for all possible normal stresses, cover the sphere. The author studies the properties of these quartics, distinguishing between the cases when the stress quadric itself is an ellipsoid or a pair of hyperboloids. The results are purely geometrical, and apply to any symmetric dyadic; in particular, the author applies them also to the state of strain.

In the case of an isotropic elastic solid, the quadrics of stress and strain, at each point, have the same principal axes and the same circular sections; hence, they have the same quartics upon the unit sphere. The author shows that, conversely, if the quartics of these two dyadics are the same, it follows that they have the same principal axes and circular sections, and are related by a generalization of Hooke's law, the principal components of the one being linear functions of the corresponding principal components of the other, with coefficients which are arbitrary functions of the invariants. C. A. Truesdell, USA

Experimental Stress Analysis

(See also Rev. 632)

579. Raymond D. Mindlin and Lawrence E. Goodman, "The optical equations of three-dimensional photoelasticity," *J. appl. Phys.*, Jan. 1949, vol. 20, pp. 89–95.

In this paper Maxwell's electromagnetic equations are applied to two- and three-dimensional photoelasticity, and the results are compared with Neumann's equations. It is shown that the customary kinematic analysis of the propagation of light through two-dimensional models is exact only for homogeneous stress. The nature and order of magnitude of the terms which result from variation of stress in the plane of the model are investigated first, and the error in neglecting these terms is demonstrated to be negligible.

Ignoring them simplifies the subsequent study of wave propagation through a truly three-dimensional model. Here, as in the two-dimensional case, the order of the differential equations for

the components of the magnetic vector is reduced by considering only forward traveling waves. However it is now necessary to neglect a term involving the product of the rate of rotation of secondary principal axes along the wave normal and the relative retardation per unit length.

The two simultaneous first-order differential equations for the components of the magnetic vector along the secondary principal axes are much easier to handle than Neumann's equations. Reduction to canonical form shows that the ratio of the rate of rotation to the retardation per unit length along the wave normal in the absence of rotation is the controlling variable.

D. C. Drucker, USA

580. **Abner Brenner and Seymour Senderoff, "A spiral contractometer for measuring stress in electrodeposits,"** *J. Res. nat. Bur. Stands.*, Feb. 1949, vol. 42, pp. 89–104.

Instead of using thin flat strips and measuring their curvature in the usual way to determine the stresses produced in coatings during electrodeposition, the authors have designed an instrument using a metal helix, in which the change in radius of curvature is read directly on a dial with tenfold magnification. The instrument can be calibrated, and errors from many sources minimized. The authors present checks with stresses determined by other methods.

A. J. Durelli, USA

581. **Abner Brenner and Seymour Senderoff, "Calculation of stress in electrodeposits from the curvature of a plated strip,"** *J. Res. nat. Bur. Stands.*, Feb. 1949, vol. 42, pp. 105–123.

Stress developed when a coating is deposited on one side of a thin plate can be calculated by measuring the curvature produced in the plate by these stresses. The authors review some old formulas for this stress and derive some new ones, taking into consideration several variables affecting the phenomenon, and in particular the difference in moduli of elasticity of the coating and of the base metal. Attention should be called to one of the formulas (49) giving the curvature as a function of the sagitta, which should presumably be $1/r = 2z/[(B/2)^2 + Z^2]$. Poisson's ratio is assumed to be the same for both materials in all the formulas.

A. J. Durelli, USA

582. **G. Hofmann, "The pressure distribution under bearing plates (Die Druckverteilung unter Lagerplatten),"** *Bauplan. Bautech.*, Nov. 1948, vol. 2, pp. 313–314.

The author studies systematically by photoelastic means the distribution of stresses in semi-infinite plates compressed in their plane over a finite width of the free edge. The load is applied through blocks of varying heights and moduli of elasticity, and for each case the isochromatic lines are drawn. A table and diagrams give the best height-to-width ratio of the block to obtain the smallest stress concentration. The paper should be useful to foundation designers.

A. J. Durelli, USA

583. **B. Sugarman, "A photoelastic approach to stress modifications caused by inhomogeneities,"** *Inst. Metals Monogr. Rep. Ser.*, no. 5, 1948, pp. 281–288.

The author describes a technique used to investigate stress concentrations due to inhomogeneities by means of photoelastic beam models. The inhomogeneities in the specimens tested were produced by drilling holes of various shapes.

Experiments were carried out first with a two-dimensional sensitive plastic known as CR.39, and afterwards with Catalin Clear Glass 800 suitable for three-dimensional investigation. A full analysis of the stress values is not treated in detail in the paper. However it seems to be possible in many cases, although asymmetric systems may introduce serious complications.

Enrico Volterra, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 593, 610, 626, 636)

584. **Daniel Bouliau, "Strength of thick tubes under internal pressure and the practical choice of the thickness of lead pipes (La résistance des tuyaux épais à la pression intérieure et la détermination des tuyaux de plomb),"** *Tech. mod. Constr.*, June 1947, vol. 2, pp. 67–71.

The author gives a new formula for the strength of a thick tube (which seems erroneous to the reviewer, as the compatibility condition is not satisfied) and compares his result with the known formulas of Lamé and Winkler. He then deduces from this comparison an empirical formula which is well suited to practical purposes. Finally, the influence of temperature and the addition of antimony on the strength of lead pipe are examined and the empirical formula is generalized to include these influences.

Ch. Massonnet, Belgium

585. **Stanley U. Benscoter and Myron L. Gossard, "Matrix methods for calculating cantilever-beam deflections,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1827, Mar. 1949, pp. 1–58.

The author presents a matrix method for calculating torsional and bending deflections of a nonuniform cantilever beam with arbitrary load distribution. The actual load distribution is replaced with an equivalent set of concentrated loads spaced at equal intervals along the beam. These concentrated loads are treated as the components of a vector.

The integrations which give shears, bending moments, etc., in the scalar problem are identified with products of matrices derivable from the structural properties of the beam. Improved accuracy is obtained with the introduction of weighting matrices based on the approximation of the load curve by a series of second-degree parabolic arcs. The adaptation of influence coefficients to the matrix methods is shown to be advantageous. The merits of the methods described are discussed by comparing the results of applying them to the free vibration of a uniform cantilever beam.

K. E. Bisshopp, USA

586. **Ned L. Ashton, "Prestretching increases strength of steel T-beams in University of Iowa tests,"** *Civ. Engng. N.Y.*, Mar. 1949, vol. 19, pp. 42–43.

The author tested beams of structural steel under pure bending. Some of the beams were prestretched up to 7 per cent of the original length, these beams and supported loads up to 50 per cent higher than the load supported by beams made of the same steel without prestretching. The author claims that present standard requirements for ductility in the structural grades of steel are not the most desirable ones, and that greater efficiency could be obtained if the yield point were closer to the ultimate strength. No discussion is given however, of the effect that reduced ductility would have on stress concentration and fatigue.

A. J. Durelli, USA

587. **J. C. W. Humfrey, "Stresses induced by the shot-peening of leaf springs,"** *Inst. Metals Monogr. Rep. Ser.*, no. 5, 1948, pp. 189–193.

This paper is one of the contributions to the Symposium on Internal Stresses in Metals and Alloys organized by The Institute of

Metals, London, October 1947. During 1943-1946, a series of experiments was carried out to determine the conditions of shot-peening which would give the maximum fatigue resistance to vehicle leaf springs. Measurements of the surface compressive stresses induced by peening were obtained by observing the change of camber of the leaves after removal of successive surface layers by solution in hot dilute sulphuric acid. All shot-peened samples showed a fatigue life of at least 200 per cent above that of untreated leaves. It was also found that the maximum stress values ranged from 35 to 60 tons per sq in. and the depth of the compressively stressed layer from 0.006 to 0.16 in.

S. K. Ghaswala, India

588. H. Gerodolle, "Longitudinal forces in conduits caused by hydraulic pressure and temperature variations (Efforts longitudinaux provoqués dans les conduites forcées par la pression hydraulique et les variations de température)," *Houille blanche*, May-June 1947, vol. 2, pp. 205–208.

Longitudinal forces in strained conduits built in at both ends may be important, not only due to temperature changes, but also because of the longitudinal contraction or expansion caused by internal pressure. Deformations and longitudinal forces due to hydraulic pressure add algebraically to deformations and longitudinal forces due to temperature variations.

The most common cases are studied: (1) Temperature variations in: (*a*) pipe with one free end; (*b*) pipe with both ends built in. (2) Hydraulic pressure in: (*a*) pipe with slip joints; (*b*) pipe with one end closed but free to move; (*c*) pipe with both ends built in. The general case, with consideration given to both the temperature and the pressure effects, is explained and an example is given. Several conditions must be examined: empty pipe at different temperatures; full pipe with outlet open; pipe with water under pressure. Furthermore, the flexibility of a pipe may be different in extension and compression.

André L. Jorissen, Belgium

Plates, Disks, Shells, Membranes

(See also Revs. 596, 609, 648)

589. Karl Girkmann, "Plates structures (Flächentragwerke)," Springer-Verlag, Wien, 1948. Paper, 9.50 × 6.25 in., 502 pp., 273 figs., approx. $12.73.

This is the second edition of an introductory treatise on plates and shells, first published in 1946. Following a brief exposition of the elements of three-dimensional theory of elasticity, the author deals extensively with the plane-elasticity problem. The technically important solutions in Cartesian and polar coordinates are developed via the Airy-function approach. This chapter contains several recent investigations which are not available in other treatises on the subject. Reflecting the author's own research interests, considerable emphasis is placed upon solutions in Fourier-integral form.

A second major section is devoted to the classical small-deflection theory of thin flat plates. In this connection a detailed discussion of the method of singularities, based primarily on the work of A. Pucher, is noteworthy. A treatment of associated stability questions is included. Two further chapters are concerned with the membrane and flexure theory of thin shells, and here again the stability problem receives a treatment which takes into account the results of recent investigations. A short concluding chapter deals with structures composed of flat-plate elements, and is motivated by problems arising in reinforced concrete design. Finally, an appendix on orthotropic plates has been incorporated in the current edition.

Throughout the book numerous applications are systematically presented, and each chapter is followed by a comprehensive and relatively up-to-date bibliography. In a desire to render this treatise accessible to students and practicing engineers, the author has avoided subjects which require an extensive mathematical background, and has occasionally confined himself to intuitive proofs and arguments. It might be pointed out that the integral (100) on p. 47 does not exist, and that the integral on p. 109 is divergent at the lower limit.

E. Sternberg, USA

590. V. Z. Vlasov, "Some new problems on shells and thin structures," *Nat. adv. Comm. Aero. tech. Memo.*, no. 1204, Mar. 1949, pp. 1–46 (transl. from *Bull. Acad. Sci. USSR Ser. tech. Sci.* (*Izv. Akad. Nauk SSSR Ser. tekh. Nauk*), 1947, no. 1, pp. 27–51).

This is an English translation of the paper reviewed previously [see Rev. 253, Feb. 1948].

591. G. W. Watts and W. R. Burrows, "The basic elastic theory of vessel heads under internal pressure," *J. appl. Mech.*, Mar. 1949, vol. 16, pp. 55–73.

Formulas are presented for the stresses and displacements in semi-infinite cylindrical shells with hemispherical, ellipsoidal, conical, torispherical, toriconical and flat heads. General solutions of the differential equations are presented without derivation, some being taken from available papers while others are taken from unpublished sources. The solution of the boundary equations for the constants of integration is illustrated in detail, using matrix notation. All final formulas are presented in matrix form. Solutions of the differential equations for an ellipsoidal head are not given, since the known solution, which is in series form, does not converge at a practicable rate.

It is recommended that the published formulas of the paper be used to prepare graphs of critical stresses in terms of dimensionless parameters. A numerical example is presented, as well as an extended bibliography.

Stanley U. Benscoter, USA

592. Otto R. Jäcker, "Calculation of circular disks (Kreisplattenberechnung)," *Bautechnik*, Nov. 1948, vol. 25, pp. 256–260.

The author attempts to show that the moments in circular plates obtained from the standard theory (see for example Timoshenko, *Theory of Plates and Shells*) are inconsistent with those in square plates. The author's formula for square plates, however, differs from that in the above reference. The application of the standard formula eliminates the inconsistency. The modifications of the theory suggested by the author appear unwarranted.

Karl Arnstein, USA

593. G. F. C. Rogers, "Stresses in turbine rotors of disc construction," *Engineering*, Mar. 1, 1949, vol. 14, pp. 121–122.

The author has demonstrated elsewhere that rotors for axial compressors which are built up from separate disks are lighter than the normal drum construction. He suggests now a new method of constructing such a rotor out of solid disks, involving welding the rims of successive disks together and eliminating the shaft. This should give an even lighter construction than one with disks mounted on a shaft, since the disks have no central hole and hence no thickening at the hub is required.

This article describes a method of stressing rotors of the proposed new type, and is an extension of a work already published ["The estimation of stresses in turbine-disc rims," *Engineering*, 1948, Jan. 2 and 9].

The boundary conditions of the problem are first defined, and then the simple case of a rotor without external loading and with parallel-sided disks is considered in detail. Four graphs of the maximum stresses in the rim are given, and the author also indicates the effect on these stresses of changing some dimensions of the system. Ch. Massonnet, Belgium

594. **Maria Esslinger, "On the calculation of sheet metal road slabs (Zur Berechnung von Flachblechfahrbahntafeln),"** *Bautechnik*, Oct. 1948, vol. 25, pp. 222–224.

The author considers the cylindrical bending of a uniformly loaded rectangular plate. Two opposite edges are either simply supported or built-in, and are not free to move toward each other. The solutions are essentially the same as those presented elsewhere [S. P. Timoshenko, *Theory of Plates and Shells*, McGraw-Hill, 1940, pp. 4–17]. G. E. Hay, USA

595. **A. Holmberg, "Tests with circular plates,"** *Acta Polyt.*, 1947, no. 3, pp. 1–110.

A very detailed account is given of tests on circular plates performed at the Royal Technical University of Stockholm. The test specimens were two steel plates one meter in diameter and approximately 0.4 cm and 0.8 cm in thickness. The loading conditions included uniformly distributed edge moments (simulated by closely spaced cantilevers actuated by wires passing over pulleys), concentrated lateral loads at the center of the plate, combinations of these two, and also single noncentral lateral loads. Central deflections, edge slopes, bending stresses and shearing stresses were determined from measurements of the rotations of mirrors attached to the surfaces of the plates.

The results, which were evidently limited to central deflections less than twice the plate thickness, were in good agreement with solutions of von Kármán's differential equations where such solutions were available. S. B. Batdorf, USA

Buckling Problems

(See also Revs. 589, 590)

596. **S. B. Batdorf and John C. Houbolt, "Critical combinations of shear and transverse direct stress for an infinitely long flat plate with edges elastically restrained against rotation,"** *Nat. adv. Comm. Aero. Rep.*, no. 847, 1946 (issued in 1948), pp. 1–8.

A solution is given of the problem of buckling of a long thin rectangular flat plate, due to combinations of compression and shear in the plane of the plate. The shear is uniform round the rectangular boundary. The compression is applied to the long edges only, its direction being parallel to the short ones. The long edges are elastically restrained against rotation. These conditions arise, approximately, in the strip of wing skin between two closely spaced chordwise stiffeners. Two independent solutions are found, an exact one from the differential equation, resulting in a transcendental stability criterion, and an approximate energy solution.

Curves are given showing the combinations of compression and shear which are critical. They differ substantially from previously proposed "interaction" formulas, the latter appearing decidedly conservative. They show that, contrary to the natural expectation that shear will render a smaller compression critical, an appreciable fraction (one third to more than one half) of the critical stress in pure shear may be applied without causing any reduction in the transverse compressive stress required for buckling. This is established by considering the limiting form of the solution for long longitudinal wave lengths, the case of compression without shear corresponding to infinite wave length. Higher shears produce the expected reduction. J. N. Goodier, USA

597. **Henry Favre, "Study of the influence of weight on the stability of a rectangular plate (Étude de l'influence du poids propre sur la stabilité d'une plaque rectangulaire),"** *Schweiz. Bauztg.*, 1949, vol. 67: Jan. 15, pp. 34–35; Jan. 22, pp. 57–58.

An analysis is given of the influence of weight on the stability of an unstiffened simply supported rectangular plate in a vertical position, the two horizontal edges of which are of length a and are subjected to uniform compressive load. The critical loads correspond to the roots of an infinite set of independent infinite determinantal equations; this set corresponds to the set of normal displacements $\sin(m\pi x/a)\Sigma_{n=1}^{\infty} a_{mn} \sin(n\pi y/b)(m = 1, 2, \ldots)$, and the smallest critical load always occurs with $m = 1$. In the range $b/a = 0$ to 5, $\gamma ha^2b/\pi^2D = 0$ to 10 (where γ is the specific weight) sufficiently accurate numerical values of the critical stress coefficients are given by finite determinantal equations corresponding to at most four consecutive coefficients a_m. The normal deflection is shown for the cases $b/a = 0.5, 1.0, 1.5, 2.0$, $\gamma ha^2b/\pi^2D = 4.0$

The author suggests various extensions of the analysis, including attention to other types of edge support and loading; the reviewer suggests that the effect of stiffening would be of interest. These extensions are analytically straightforward but in general the computational work would be greater than for the case studied; some extensions would involve the preliminary problem of the determination of the initial plane stress distribution.

H. G. Hopkins, England

598. **F. Stüssi, "Calculating the buckling stresses of compressed rectangular slabs (Berechnung der Beulspannungen gedruckter Rechteckplatten)"** (with summaries in French and English), *Publ. int. Ass. Bridge Struct. Engng.*, 1947, vol. 8, pp. 237–248.

A rectangular thin plate is considered under compressive forces acting on two opposite edges. Following Levy [*C. R. Acad. Sci. Paris*, 1899, vol. 129, pp. 535–539] the deflection is assumed to be of a form which reduces the problem to the solution of an ordinary differential equation of fourth order. The usual types of support of the loaded edges are considered: (*a*) simply supported; (*b*) clamped; and (*c*) free edge. The resulting boundary-value problem is solved by an iterative method employing finite differences.

Three examples are considered in detail: (*a*) linearly varying edge force with simply supported edges; (*b*) uniform compressive force with clamped edges; and (*c*) uniform compressive force with one edge free, one simply supported. Comparison with known results shows that the convergence is quite rapid.

William Pell, USA

599. **Giles G. Green, George Winter, and T. R. Cuykendall, "Light gage steel columns in wall-braced panels,"** *Cornell Univ. engng. Exp. Sta. Bull.*, no. 35, part 2, Oct. 1947, pp. 3–50.

The method of Klemperer and Gibbons for the analysis of columns with intermediate elastic supports is applied to an investigation of the buckling of light steel columns or wall studs when the elastic support is provided by wallboard materials. Buckling is assumed to take place in the direction of the wallboard. Experimental values are given for the equivalent spring constant of several wallboard materials. Since these values have been obtained essentially by a tensile test of a strip of wallboard of length equal to the column spacing, they are apparently valid only for the case of buckling of a single column and are not

applicable to the general instability of the wall panel where the significant deformations may be the shearing deformations of the panel.

The analysis is extended to the case of a column with initial eccentricity. In the reviewer's opinion incorrect formulas have been used in the calculation of the internal and external energy, so that a further study seems to be warranted. However, experimental results are given which seem either to verify the formulas developed for the buckling strength or to indicate an insensitiveness to initial eccentricities. John E. Goldberg, USA

600. George J. Heimerl and William M. Roberts, "Determination of plate compressive strengths at elevated temperatures," *Nat. adv. Comm. Aero. tech. Note*, no. 1806, Feb. 1949, pp. 1–20.

Compressive stress-strain tests and local instability tests were made on extruded *H*-sections of 75S-T6 aluminum alloy at temperatures up to 600 F. Results show that valid methods for computing the critical compressive stress of plate elements at room temperature are applicable at elevated temperatures if the proper stress-strain curve is used. For this particular material in the plastic range, the secant-modulus method proves to be slightly unconservative at elevated temperatures.

John E. Goldberg ,USA

601. William A. Hickman and Norris F. Dow, "Data on the compressive strength of 75S-T6 aluminum-alloy flat panels with longitudinal extruded *Z*-section stiffeners," *Nat. adv. Comm. Aero. tech. Note*, no. 1829, Mar. 1949, pp. 1–21.

Results are presented for flat-ended compression tests of 75S-T6 aluminum-alloy flat panels reinforced with extruded *Z*-section stiffeners. Sheet thicknesses were 0.040, 0.064 and 0.102 in.; all stiffeners were 0.102 in. thick, had a depth of 1.5 to 4 in., and were spaced at from 15 to 40 times the sheet thickness. Abnormal values of critical stress were noted for extremely short specimens and are attributed to end restraints. John E. Goldberg, USA

602. W. K. G. Floor, "Experimental investigation of the post-buckling behaviour of stiffened, flat, rectangular plates under combined shear and compression. Part II" (in English), *Nat. LuchtLab. Amsterdam Rap.*, no. S.328, Dec. 30, 1947, 41 tables, 68 figs., 45 pp.

This report describes the results obtained in an experimental investigation into the diagonal tension field in stiffened flat plates loaded by a combination of shear and compression parallel to the shorter edges of the individual panels; it forms a continuation of the work described in Part I of the report under the same title [*Nat. LuchtLab. Amst. Rap.*, no. S.300. See Rev. **1196**, Aug. 1948]. The plates had five panels, each of which had a length to width ratio a/b of 3.16. The ratios of plate thickness to panel width t/b used in the tests were $^1/_{357}$ and $^1/_{118}$. Tests on plates for which t/b was $^1/_{167}$ were described in Part I.

An extension of the theory of W. T. Koiter [*Nat. LuchtLab. Amst. Rap.*, no. S.295] is developed and compared with the tests, with generally reasonable agreement. New proposals for practical applications are made, and illustrated by means of a numerical example. P. S. Symonds, USA

603. Riccardo F. Baldacci, "On buckling load in the elasto-plastic range (Sul carico di punta in periodo elasto-plastico)," *G. Gen. civ.*, Oct. 1948, vol. 86, pp. 571–574.

In an attempt to improve the mathematical aspect of the theory of short columns of rectangular cross section due to F. R. Shanley [see Rev. **72**, Jan. 1948], the author assumes a deflection curve of sinusoidal form, similar to the one derived in the Euler theory. The transition to a bent equilibrium position requires a certain increase of the axial load and a displacement of the neutral axis, whereby the tangent modulus is assumed to hold on the concave and the elastic modulus on the convex side of the column. To the reviewer the latter assumption seems to be a misinterpretation of Shanley's theory; neither was he able to follow the author's conclusions as to the limiting properties of the column load thus derived. Folke K. G. Odqvist, Sweden

604. L. H. Donnell, "The critical axial compression or tension on a bar, for all possible positive and negative end fixities," *Reissner Anniv. Vol.*, J. W. Edwards, Ann Arbor, 1949, pp. 183–196.

A solution is given for the classical problem of the buckling of a uniform column with ends having equal or unequal elastic rotational restraints. In addition to the transcendental equation giving the exact solution of the differential equation, the relation between the end fixities and the buckling coefficient is given graphically for all positive and negative values of end fixities for both first- and second-mode buckling, and a simple approximate algebraic expression is given for this relation for first-mode buckling. Applications to simple frame and truss problems are given. The author does not claim that this general solution will make practical the direct solutions of such problems as the buckling of a complicated truss, but does anticipate that its presentation will help to clarify the more complex problems and suggest new methods of approach. S. B. Batdorf, USA

Joints and Joining Methods

605. G. M. Monroe, "Analysis of eccentrically loaded riveted or bolted joints," *J. aero. Sci.*, Feb. 1949, vol. 16, pp. 120–122.

A semigraphical procedure is presented for computing the resultant shear stresses in the bolts or rivets of a joint based on the usual assumptions of elastic action. On a scale drawing of the joint a point is located whose distance from each rivet is multiplied by a constant factor to determine the stress on the rivet. The coordinate of this point and the multiplying factor are computed from equations involving the load on the joint, the total rivet area and the polar moment of inertia of the rivet areas. (The analysis assumes perfect fit of all rivets and perfect register of rivet holes.)

T. J. Dolan, USA

606. D. Rozet, H. C. Campbell, and R. D. Thomas, Jr., "Effect of weld metal composition on the strength and ductility of 15% Cr-35% Ni welds," *Weld. Res. Suppl.*, Oct. 1948, vol. 13, pp. 481–491.

The tendency of 15 per cent Cr-35 per cent Ni alloy when arc-welded with coated electrodes is to develop difficulties occasionally because of low ductility and cracking of the weld metal.

Using electrodes having a lime-type coating, and varying the weld composition through coating additions, it is shown that the amount of carbon and silicon in the weld metal greatly influences the mechanical properties and the soundness of the welds. The optimum for 15 per cent Cr-35 per cent Ni weld-metal composition is about 0.20 per cent carbon and 0.40 per cent silicon, with other elements present in conventional amounts. The strength increases with increasing carbon content, and decreases with increasing silicon content. The ductility increases with carbon and with silicon as long as these are below the optimum value, and then it decreases.

Sulphur and phosphorus beyond the usual commercial limits of

about 0.25 per cent have been proved harmful. Manganese has no apparent influence. Columbium is believed to be unnecessary since the ductility is substantially reduced by its addition.

The exact cause of fissures has not yet been clearly established, but the evidence points to the existence of intergranular slag films present in the high-silicon and low-carbon alloys. Attempts to show that fissures could be eliminated by quench-annealing heat treatments were unsuccessful, thus confirming the theory that the grain-boundary microfissures form during the solidification of the metal. M. W. Seese, USA

607. J. A. **Wheeler, "The stress system causing hard-zone cracking in welded alloy steels,"** *Inst. Metals Monogr. Rep. Ser.*, no. 5, 1948, pp. 367–373.

A discussion is given of the problem of crack formation in the heat-affected zone adjacent to an arc weld made between alloy-steel plates. It is shown that such cracks start with the formation of microcracks which subsequently extend into a major crack under the influence of the main welding stresses. The microcracks are produced by the combined action of macro- and microstresses, the latter due to the strains accompanying the transformation from austenite to martensite and in some cases also to the development of hydrogen gas pressure in internal rifts. R. L. Bisplinghoff, USA.

Structures

(See also **Revs. 582, 586, 588, 589, 591, 594, 644, 648**)

608. **"Royal Aeronautical Society Data Sheets. Vols. 1 and 2, Stressed skin structures; Vol. 3, Aerodynamics,"** Royal Aeronautical Society, London, 1948. Loose-leaf binders, 11.25 × 10.25 in., 191 total sheets, approx. $65.

These data sheets of the Royal Aeronautical Society are intended primarily to expedite and simplify the work of aircraft designers by presenting, generally in graphical form, information of the type required in routine problems of analysis. The sheets are grouped at present into three loose-leaf volumes—one containing 68 sheets of aerodynamics data, and two volumes containing 123 sheets of data on stressed skin structures. A fourth volume for use in performance calculations is said to be in preparation.

The aerodynamics volume presents such typical data as: properties of the atmosphere; variation of stability derivative with dihedral, sweepback and other geometric parameters; slope of lift curve for wing and empennage; drag of streamline bodies of revolution; and lift and drag coefficient for various flap arrangements. The structural-data sheets present information on alloys, flexural and torsional properties of the common shapes of sections, stability of struts and plates, and an extensive section on shear-lag or diffusion phenomena in reinforced sheets.

Each sheet is complete in itself, presenting the design chart on the face and, on the reverse side, notation, explanation, notes and references for derivation, and an illustrative example. Provision is made for keeping the material up to date by the issuance of new or revised pages at desirable intervals.

John E. Goldberg, USA

609. Frank E. **Richart, "Reinforced concrete wall and column footings—Part 2,"** *J. Amer. Concr. Inst.*, Nov. 1948, vol. 20, pp. 237–260.

In this second part of his report [see Rev. **1623**, Nov. 1948] the author describes and analyzes results on three more series of tests made on concrete footings, supported on a bed of steel car springs to simulate soil pressure and loaded in a large hydraulic testing machine. Tests of Series No. 3 and No. 4, made on 7-ft square footings, were planned to study the failure of footings in bond, while tests of Series No. 5, made on 6 rectangular footings 6 × 9 ft in plan and on 6 rectangular footings 5 × 10 ft in plan, all having a 14-in. square stub in the center, were planned to study failure by tension.

The most important conclusions are: (1) Stresses in the reinforcing steel, derived from the measured strains, agree near the yield stress with the values of stresses computed from the full statical bending moment acting on the footing. (2) The behavior of rectangular footings does not differ greatly from that of square ones. (3) The use of a pier between footing and column is found to be effective in increasing the load capacity of the footing and in saving reinforcement. (4) The results generally are in good agreement with design theory. Enrico Volterra, USA

610. **Giacinto Turrazza, "Arched beams in framed structures (Der gekrümmte Balken als Teil von Rahmentragwerken),"** *Bautechnik*, Sept. 1948, vol. 25, no. 9, pp. 193–197.

This analysis of arched beams or girders in framed structures results in equations from which the end moments on columns and beams in single- or double-span frames may be computed. If tie rods are used between the ends of the arched members the loads in them may be obtained also.

The development may be followed readily by one familiar with the slope-deflection relations and the method of elastic loads, and the resulting equations are less cumbersome than those commonly encountered in the analysis of such frames. Terms which differ for different types of loading are given for usual types of loading. Solutions for two examples are included, carried to the point of setting up the algebraic expressions. A third example is solved numerically, the effect of a uniformly distributed load being considered first, and that of a twenty-degree rise in temperature second. Joseph S. Newell, USA

611. **M. Fine and D. Williams, "Stress concentration due to four-point fixing at front end of monocoque fuselage, theoretical analysis,"** *Rep. Memo. aero. Res. Counc. Lond.*, no. 2100, Apr. 1941 (issued in 1948), pp. 1–28.

The transfer of concentrated loads from four points of attachment is investigated by both rigorous and approximate methods. Practical conclusions are presented and results compare favorably with test data.

The methods developed will be useful to designers in determining the proper rate of taper for the longerons and reinforcement of the skin near the points of attachment. F. R. Shanley, USA

612. **Waclaw Olszak, "The effect of transverse grate reinforcement in compressed concrete elements (L'effet du frettage en nappes transversales des constructions en béton armé)"** (with English and German summaries), *Publ. int. Ass. Bridge Struct. Engng.*, 1948, Third Congress, prelim. publ., pp. 247–266.

This paper deals with theoretical considerations on the behavior of concrete columns reinforced with transverse steel grates. Formulas and diagrams are presented which give the increase in longitudinal loading capacity of the column as a function of the properties of the concrete and steel and the percentage of transverse reinforcement. Comparisons are made with some previous tests and with concrete specifications. The advantage of this method of construction in relation to the more conventional spirally reinforced concrete has not been made apparent.

A. J. Durelli, USA

613. Linton E. Grinter, "Theory of modern steel structures," Macmillan, New York, 1949, vol. 2. Cloth, 9.4 × 6.2 in., 312 pp., 224 figs.

In revising this textbook, the major aim of the author has been to improve the readability and teachability of the text by clarifying many of the derivations and by some rearrangement of material. A new section has been added on shear flow in semimonocoque cylindrical shells and a brief discussion of aerodynamic oscillations of suspension bridges is also included in this edition. John E. Goldberg, USA

614. Ling-Hi Tsien, "A simplified method of analyzing suspension bridges," *Proc. Amer. Soc. civ. Engrs.*, Sept. 1948, vol. 74, pp. 1093–1128.

This paper offers a valuable simplification in the practical computation of suspension bridges. The type of bridge discussed comprises a stiffening truss divided into three simply supported girders, one for the main span and the others for the side spans; all three are supposed to be suspended from the cable, but the theory applies equally well to a bridge with straight backstays.

The paper is divided into three parts. In the first one, the author computes the additional horizontal component H of cable tension induced by the live load. He draws the following important conclusion from this study: when the live load moves along the bridge, the horizontal tension in a cable varies very nearly according to certain circular functions, irrespective of both the ratio between the dead load and the live load and the degree of stiffness of the truss. The author then gives a table summarizing the expressions for H for various loading conditions useful in design. These expressions are exceedingly simple in form and are as accurate as the results computed by the deflection theory.

In the second part of his paper, the author utilizes the result obtained above to reduce the expressions for moments and shears in the stiffening truss to explicit and simple formulas, containing only hyperbolic functions. Moment formulas are given in a table for the principal live-loading conditions which need to be considered in design. The third part is devoted to the determination of maximum moments, shears, slopes and deflections in the stiffening truss. On the basis of simple expressions established in the second part, the author undertakes a direct analytical determination of these maxima, for the eight cases of loading and temperature to be considered in the design computation.

Finally, as an illustrative example, the method is applied to Mount Hope suspension bridge in Rhode Island. The complete analysis required only 6 to 8 hr. All numerical values were calculated on a 12-in. slide rule and the results, when compared with those obtained by the deflection theory, show complete agreement within one per cent. Ch. Massonet, Belgium

615. P. P. Loupman, "A new method of calculation for hydraulic structures of lightly reinforced concrete," *Civ. Engng. Lond.*, Jan. 1949, vol. 44, pp. 34–36.

This article is an abridged translation of a paper in the August 1947 issue of the Russian journal, "Gidrotekhnicheskoye Stroitelstvo." The claim is made that more than half of all hydraulic structures are built of lightly reinforced concrete, because there is a weight saving in comparison with structures of mass concrete, and because there is an avoidance of hair cracks in comparison with heavily reinforced concrete under limiting tensile loads.

The existing standard specifications require that lightly reinforced concrete satisfy the same factors of safety for the formation of cracks and for ultimate strength as mass concrete; this leads, in effect, to a doubling of the factors of safety. The author proposes (plotting curves for illustration) that only at the mass concrete extreme should the safety factors with less and less reinforcing material match those for no reinforcement, and that only at the other extreme should the safety factors match those in the standard specifications for regular reinforced concrete. The problem, of course, is to determine the intermediate safety factors.

The subject of lightly reinforced concrete will be fully described in the author's book now being prepared for printing; this article gives only fundamental data.

W. C. Johnson, Jr., USA

616. Alfred S. Niles, "Overbalancing in residual-liquidation computations," *Nat. adv. Comm. Aero. tech. Note*, no. 1457, Feb. 1949, pp. 1–80.

This paper is an application of Southwell's relaxation method to statically indeterminate structures. By developing methods for the systematic selection and use of group displacements, the liquidation of residuals is expedited. Thus the amount of labor needed to obtain the desired result can be greatly reduced in many practical cases. The application of the method is illustrated by numerical examples. Enrico Volterra, USA

617. N. J. Hoff, V. L. Salerno, Harold Liebowitz, Bruno A. Boley, and Sebastian V. Nardo, "Concentrated load effects in reinforced monocoque structures," *Reissner Anniv. Vol.*, J. W. Edwards, Ann Arbor, 1949, pp. 277–332.

This is a symposium of five articles by each of the five authors mentioned above. These authors report on their investigations of the distribution of concentrated loads between the rings and the sheet-stringer structure of circular shells. One of the articles deals with experimental investigations, the others cover theoretical work some of which has appeared in previous publications. The general results show that the distribution of shear loads between the ring and the shell cannot be predicted correctly from elementary beam theory and that large errors in ring analysis result if localized effects are neglected. This was originally shown by Wignot [*Nat. adv. Comm. Aero. tech. Note*, no. 929].

F. R. Shanley, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 571, 574, 627, 630, 631, 633, 649)

618. Charles Mack, "Plastic flow, creep, and stress relaxation, Part IV. Anomalous flow as an order-disorder transition," *J. appl. Phys.*, Nov. 1948, vol. 19, pp. 1082–1091.

In plastic systems at rest, steric hindrance prevents molecules from occupying certain positions and introduces a certain degree of orientation or structure. Under shear stress many more positions are forbidden to a molecule in motion because of the free-rotation requirement. To make more positions available, the system must increase its volume under stress in accordance with Eyring's "hole theory" of viscosity; simultaneously it changes from a state of greater order to one of greater disorder. (The author's concept of order may be exemplified by a truss in which all component members are in their right places, as against an apparent order where the different members are floating downstream in a parallel arrangement.)

Based on the order-disorder and hole concepts, equations are developed which present the strain rate at constant temperatures as functions of stress, volume change, and degree of order-disorder. The changes in entropy and interaction energy accompanying the strained state are expressed by a generalized partition function. The theory is applied to plastic flow, stress hard-

ening, work hardening, strength of plastic systems and viscoelastic effects. The latter are exemplified by data on polychloroprene under tension. Hans F. Winterkorn, USA

619. **L. R. Jackson, K. F. Smith, and W. T. Lankford, "Plastic flow in anisotropic sheet steel,"** *Metals Technol.*, Aug. 1948, vol. 15, T.P. no. 2440, pp. 1–15.

A method is proposed for comparing the stress-strain curves obtained under different states of combined stress in an anisotropic metal. The yield criterion and stress-strain relations which are developed are equivalent, in the special case treated in the paper, to those given by R. Hill [*Proc. roy. Soc. Lond. Ser. A*, May 26, 1948, vol. 193, pp. 281–297; see Rev. **1121**, July 1948]. It is assumed further that the work hardening is a function only of the total plastic work, and that the state of anisotropy does not change in the range of strain considered. The initial state of anisotropy is specified by the measured values of the strain ratios in tension tests along and across the rolling direction.

On this basis, the "effective" stress-strain curve for steel sheet under biaxial tension in a bulge test is compared with the uniaxial stress-strain curve. The agreement appears to be satisfactory. Rodney Hill, England

620. **A. S. Nowick and E. S. Machlin, "Quantitative treatment of the creep of metals by dislocation and rate-process theories,"** *Nat. adv. Comm. Aero. Rep.*, no. 845, 1946, pp. 1–10.

Applying the theory of rate processes and the theory of dislocation to the creep of pure metals, the authors derive an equation for the steady-state rate of creep as a function of applied stress and temperature. An equation is first derived for the steady-state rate of creep, which shows that the rate of generation of dislocations is the primary factor which determines the variation of creep rates among materials. A specific model for the mechanism of generation of a dislocation is proposed and the energy necessary for generation of a dislocation calculated. The rate of generation can then be calculated in terms of rate-process theory as a frequency factor multipled by the probability that a source of dislocation will attain the energy required to generate a dislocation.

The form of the equation for steady-state rate of creep is in agreement with empirical equations found in the literature. The theory is also used to predict the dependence of steady-state rate of creep on physical constants of the material and good agreement is obtained with data in the literature for pure annealed metals. The rate of creep is found to decrease with increasing modulus of rigidity. B. M. Rosenbaum, USA

621. **H. W. Swift, "Plastic bending under tension,"** *Engineering*, 1948, vol. 166: Oct. 1, pp. 333–335; Oct. 8, pp. 357–359.

It is the main thesis of the paper that plastic bending of a thin wide strip under a superimposed tensile stress inevitably produces a thinning of the material. According to the author it is this thinning which is mainly responsible for the discrepancy between theory and experience in deep drawing.

Assuming that a linear relation between stress and strain (uniform strain-hardening rate) exists in the plastic range, and that lateral contraction is suppressed, the thinning of a strip, bent and advanced over a roller under a superimposed tensile stress, is calculated. In a first analysis the problem without friction is treated, and in a second step friction between roller and strip is considered.

To check the validity of the theoretical derivations and to find representative values for the coefficient of friction, an apparatus was constructed to bend a strip of metal over a roller. The materials investigated were mild steel, brass and aluminum. The calculated and experimental values were found to be in good agreement. J. H. Palm, Holland

622. **B. M. Rovinski, "On the formation of stresses of the second kind in plastic deformation"** (in Russian), *J. tech. Phys. (Zh. tekh. Fiz.)*, Oct. 1948, vol. 18, pp. 1273–1281.

The author studied by means of X rays the formation of stresses of the second kind (small-scale residual stresses with individual crystals) due to plastic deformation in tungsten crystals, annealed mild carbon steel and heat-treated steel.

He arrived at the following conclusions: (1) In isolated deformed crystals of tungsten and in crystallites of deformed steel there are present considerable stresses of the second kind. (2) The growth of stresses of the second kind in crystallites due to their plastic deformation causes their texture to become more mosaic. (3) The stresses of the second kind which arise in polycrystals due to plastic deformation are of two kinds: aggregate and mosaic. Aggregate stresses of the second kind are caused by the polycrystalline nature of the material and the anisotropy of the crystals. Mosaic stresses of the second kind are due to the mosaic texture of crystallites.

M. M. Gololobov, Czechoslovakia

623. **I. A. Oding, "Relaxation and creep of metals considering nonuniform stress distribution"** (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekh. Nauk)*, Oct. 1948, no. 10, pp. 1561–1575.

The author supposes that self-diffusion takes place in a pure metal under the influence of a stress gradient. Atoms migrate from the compressed into the dilated parts of the body. He claims that this migration explains the fact that the creep of a bent specimen is, at the beginning, about twice as great as in the tensile test, as previously shown by the author [*Mach. Building News* (*Vestnik mashinostroyenya*), 1944, no. 5]. The starting point of the calculation is the approximate relation

$$d\epsilon/dt = -(1/E)d\sigma/dt \qquad (1)$$

(ϵ is amount of creep, σ stress, and E modulus of elasticity).

The migration of a volume dq through a surface f into a dilated unit of volume results in a change of stress $\partial\sigma/\partial q = -E/(1-2\mu)$ (μ is the Poisson ratio). The author finds

$$\partial q/\partial t = -fD\partial\sigma/\partial x \sinh(\alpha\sigma/RT)$$

[α is the dimensionless atomic volume in which the elementary step of diffusion takes place; $D = A\exp(-Q/RT)$ is the constant of self-diffusion]. It follows that

$$\partial\sigma/\partial t = AL(\partial\sigma/\partial x)\sinh(a\sigma),$$

where a is α/RT, L is $2Ef[\exp(-Q/RT)]/(1-2\mu)$. Then for the surface of a bent rectangular specimen ith the height h, for small values of $a\sigma$,

$$\sigma = \sigma_* - (2\sigma_* L/h)t \sinh(a\sigma) \sim \sigma_*/(1 + 2\sigma_* Lat/h)$$

(σ_* is initial stress). It follows from Equation (1) for the creep ϵ_{pd} by self-diffusion that $\epsilon_{pd} = Cpt/(1+pt)$, where

$$p = 2\sigma_* La/h.$$

In addition to ϵ_{pd}, there is a creep component ϵ_{ps} due to gliding on the grain boundaries. It is assumed that $\epsilon_{ps} = v_p t$. The total creep is therefore

$$\epsilon_p = v_p t + Cpt/(1+pt) \qquad (2)$$

with appropriate values for p, C and v_p. Five experimental creep curves given in the paper for different steels at high temperatures could be represented extremely well by Equation (2).

The initial portion of the relaxation curve can be represented in the form

$$\sigma = \sigma_0 \exp[-kt/(1 + pt)] \quad (3)$$

if it is assumed that the velocity of relaxation is proportional to the stress [$k = \sigma_*/(1 + pt)$; σ_0 seems to be the same as σ_*]. Three experimental curves are shown to be in fairly good agreement with Equation (3). It is stated that the creep relation given by the author is the first one to be derived theoretically.

G. Masing, Germany

624. Andrew W. McReynolds, "Plastic deformation waves in aluminum," *J. Metals*, Jan. 1949, vol. 1, pp. 32–45.

A simple lever system was used for making tensile tests at a constant rate of loading, with the load maintained during yielding. The elongation was measured electrically by the use of resistance wire gages. Stair-step stress-strain curves were found in the plastic deformation of annealed 28 aluminum and Al-Cu alloys in a limited temperature range. It does not occur in 99.99 per cent aluminum. Annealed 70/30 alpha brass was found to show a similar stepped stress-strain curve.

The discontinuities were found to result from propagation of waves of plastic deformation along all or part of the specimen length; between the occurrence of such waves practically no plastic strain occurs. The type of deformation observed takes place by the appearance of slip bands, but differs from the more usual type in that slip on individual bands is not independent; strain, once initiated, spreads to adjacent grains.

The author summarized his proposed mechanism for the observed phenomenon of repeated plastic waves in 28 aluminum and Al-Cu alloys as follows: Motion on slip bands during strain results in lattice vacancies and imperfections which greatly accelerate general diffusion and precipitation of copper aggregates in the immediate vicinity. Consequent precipitation hardening stops all the relaxation on the slip bands. When the stress eventually increases enough to initiate further slip at some point, the hardened condition of the lattice causes high stress concentrations in the vicinity, which facilitate spread of the strain into adjacent grains. As successive grains break down under the stress concentrations, a wave of strain spreads through the metal and is followed by further precipitation hardening, thus starting a new cycle.

T. S. Kê, USA

625. A. M. Freudenthal and M. Reiner, "A law of work hardening," *J. appl. Mech.*, Sept. 1948, vol. 15, pp. 265–273.

The authors present a formal theory of work hardening of metals based upon the ideas of Bragg that slip will occur whenever such slip will be accompanied by a decrease in the free energy of the system. It is found that, through the use of certain further plausible assumptions, the Bragg concept leads to the type of work-hardening law which is observed.

The reader should be aware, however, that the fundamental assumption of the Bragg concept does not rest upon a firm foundation, and that the mechanism whereby slip is triggered is one of the outstanding problems in the physics of metals.

Clarence Zener, USA

Failure, Mechanics of Solid State

(See also Revs. 587, 607, 620, 688)

626. R. Beeching and W. Nicholls, "A theoretical discussion of pitting failures in gears," *Proc. Instn. mech. Engrs.*, 1948, vol. 158, no. 3, pp. 317–323.

The authors investigate by Hertz's equations the state of stress in the neighborhood of the line of contact which is caused by pressing together two cylinders with parallel axes. They reach the conclusion, already found by other authors, that the most highly stressed point in each cylinder occurs at a distance 0.786 b below the line of contact, where 2 b is the breadth of the line of contact. At a great enough pressure, therefore, fracture occurs first under the surface; this agrees with the experiments of S. Way. Repeated stresses in gear teeth thus result in the well-known small ridges called pittings. On the basis of these investigations, and in good agreement with experiments, pittings are expected to occur when the maximum pressure at the center of the line of contact is equal to or greater than the compressive yield stress of the material.

If friction forces are added to the normal pressure on the contact surface, the occurrence of pittings becomes more likely. The authors point out that the surface treatment of gear teeth must therefore be deep enough to include the regions of greatest stress, so that the softer core material is not subjected to too great a stress. The necessary thickness of the treated layer is given as a function of the strength of the core material. It is pointed out that gears of relatively small dimensions usually have the necessary thickness of the treated layer, but in larger gears this layer is frequently too thin.

Ludwig Föppl, Germany

627. F. C. Frank, "On the equations of motion of crystal dislocations," *Proc. phys. Soc. Lond. Sect. A*, Feb. 1949, vol. 62, pp. 131–134.

This paper cannot be well understood without knowledge of two papers of G. I. Taylor [*Proc. roy. Soc. Lond. Ser. A*, 1934, vol. 145, p. 362] and J. M. Burgers [*Proc. kon. Ned. Akad. Wet.*, 1939, vol. 42, p. 293] on crystal dislocations, in the first of which so-called bridge dislocation and in the second of which so-called screw dislocation is treated.

It is shown in the present paper that when a Burgers screw dislocation moves with velocity v, it suffers a longitudinal contraction by the factor $(1 - v^2/c^2)^{1/2}$, where c is the velocity of transverse sound. The total energy of the moving dislocation is given by the formula $E = E_0/(1 - v^2/c^2)^{1/2}$, where E_0 is the potential energy at rest. Taylor dislocations behave in a qualitatively similar manner, but their study is complicated by the fact that both longitudinal and transverse displacements and sound velocities are involved.

The author suppresses part of his own work and draws attention to a more complete treatment by J. D. Eshelby of this part of his problem, which will be published in the near future.

C. B. Biezeno, Holland

628. C. Gurney, "The statistical estimation of the effect of size on the breaking stress of rods," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2157, Mar. 1945 (issued in 1947), 10 pp.

Sets of strength tests on two lengths of specimens 1 in. long and 240 in. long were made. The specimens were cut from the same cord of steel wire drawn from a cast whose chemical analysis was 0.19 C, 0.121 Si, 0.0425 and 0.040 P and 0.56 Mn. The finished size was 0.0280 in.

The results of three sets of tests have been compared with those predicted by theory, assuming that a normal distribution of strength would be obtained for a unique but initially known length of wire, and that the strength of wire is statistically homogeneous.

The experimental results show small differences in distributions of strength of the long and short wires. The trend of the results is in agreement with the theory that strength and scatter of test results decrease with increasing length, and skewness tends to be-

come more negative. However, the experiments show that influences other than those which the theory covers have significant effects. Clayton O. Dohrenwend, USA

629. C. C. Hodgson and H. G. Baron, "Large crystal grain size in silicon-chromium valve steel," *J. Iron Steel Inst.*, Feb. 1949, vol. 161, pp. 81–85.

An investigation was made of the exceptionally coarse crystal grain size occasionally found in engine exhaust valves made of silicon-chromium valve steel. Experiments on heat-treating procedures indicated that ferrite was liable to remain undissolved unless the temperature of heating for hardening exceeded 1000 C. British specifications allow or require 950 C for the hardening operation; undesirable structures may remain to some extent at this temperature. Tests proved that these steels were susceptible to "cold work" while still at a bright red heat (750 to 900 C); a condition of critical strain may remain after cooling to room temperature. A recrystallization of critically strained material takes place on reheating and a structure of enlarged grains remains if the reheating temperature is below that required for completion of the $\alpha \longrightarrow \gamma$ transformation. T. J. Dolan, USA

Design Factors, Meaning of Material Tests

(See also Revs. 615, 636, 645)

630. F. V. Warnock and J. B. Brennan, "The tensile yield strength of certain steels under suddenly applied loads," *Proc. Instn. mech. Engrs.*, 1948, vol. 159, no. 37, pp. 1–10.

This paper describes the determination of the dynamic tensile yield stresses for eight steels. The dynamic loads are applied by means of the falling-weight type of impact machine and electrical resistance strain gages mounted on the specimens were used to record the loads.

A comparison of the dynamic and static yield strengths shows that under dynamic loading there is an increase of 21 to 36 per cent for the carbon steels. The heat-treated-alloy steels tested showed no appreciable increase in the yield strength with increase in rate of loading.

Comparisons are made of the test results obtained with those of other investigators, and a theory is proposed to account for the variation in sensitivity of the yield strength to load rate.

Joseph Marin, USA

631. A. F. C. Brown and R. Edmonds, "The dynamic yield strength of steel at an intermediate rate of loading," *Proc. Instn. mech. Engrs.*, 1948, vol. 159, no. 37, pp. 11–21.

This paper presents a comparison of dynamic and static yield strengths of eight steels. These steels vary from a mild steel to a heat-treated-alloy steel. The rate of loading used corresponds to that produced in a ship under the action of underwater explosion.

The dynamic yield strengths of the steels with low static strength were twenty to thirty per cent greater than the static-yield-strength values. For the stronger steels this increase was less, being negligible for the case of the heat-treated-alloy steel. The results obtained conform with those reported by other investigators. Joseph Marin, USA

Material Test Techniques

(See also Revs. 624, 630, 637, 643, 649)

632. E. H. Bucknall, W. Nicholls, and L. H. Toft, "Delayed cracking in hardened alloy steel plates," *Inst. Metals Monogr. Rep. Ser.*, no. 5, 1948, pp. 351–365.

A summary is given of an investigation into the cracking due to internal stresses of hardened alloy-steel plates and welded steel structures. It was found that the cracking phenomenon could be closely related to instances of failure of test pieces made in the form of conical Belville washers. These test pieces, which are shallow conical disks with a central hole, are pulled down against a steel plate by means of a chamfered nut on a central bolt. The stresses in such disks are calculable and they thus provide a means of applying a steady stress of known magnitude simultaneously to a large number of small test coupons. The application of this test on a statistical basis led to the recognition of the dependence of cracking susceptibility on composition and on tempering treatment. R. L. Bisplinghoff, USA

633. G. Dawance, "A new apparatus for study of hysteresis and internal friction of plastic materials subjected to alternating bending (Un nouvel appareil pour l'étude de l'hystérésis et du frottement interne des matières plastiques en flexion alternée)," *Bâtim. Trav. publics*, June 1947, no. 34, pp. 1–7.

For materials of large internal friction or damping capacity, the apparatus developed gives a complete trace of the stress-strain loop, thus permitting direct measurement of the energy loss. The test specimen is attached to a rigid support at its upper end. A relatively long heavy pendulum is attached to its lower end through a ball bearing. As the pendulum swings, the horizontal force component flexes the test specimen. A light beam shining on a mirror fastened to a triangular frame, having one corner fixed in position, one corner attached to the pendulum head, and one corner attached to the end of the specimen, allows photographic recording of the force-deformation loop. For stresses at which the damping is small the apparatus is used as an ordinary vibration decay system.

From preliminary tests on plastics the author finds that the relative damping capacity is connected with the phase angle between the pendulum force and specimen deformation by the relation $\Delta W/W = 5 \sin \varphi$, rather than $\Delta W/W = 2\pi \sin \varphi$, as it would for an elliptical loop (pure sinusoidal motion).

J. M. Robertson, USA

634. E. H. Habib, "A method of making highspeed compression tests on small copper cylinders," *J. appl. Mech.*, Sept. 1948, vol. 15, pp. 248–255.

Oxygen-free high-conductivity annealed copper cylinders were tested at David Taylor Model Basin for calibration of energy absorption-deformation relations in impact, at velocities in the range encountered when such cylinders are used in pressure gages for underwater explosions. The apparatus used for testing was an air gun that projected striking pistons of different masses at various velocities against the specimens and measured striking and rebound velocities. From these velocities the energy of deformation was determined and plotted against the measured total permanent deformation of the specimens.

Energy absorption increased at an increasing rate with deformation. The energy absorption at average strain rates of 100 to 3000 per sec ranges up to 30 per cent greater than static energy absorption for the same average deformation. While most of the specimens showed uniform deformation or a slight "barreling," some of those tested at higher velocities showed a flaring-out at the ends. A possible explanation is advanced in terms of the theory of propagation of plastic strain. Stress-strain curves derived by numerical differentiation of the energy curves are presented, and the higher speed curves are shown together with the static stress-strain curve. Comparison of these tests with constant-strain-rate tests at Carnegie Institute of Technology shows good agreement.

In addition to providing the calibration data for which they were intended, these tests add further evidence on the general question of the properties of metals at higher strain rates.

Henry A. Lepper, Jr., USA

Mechanical Properties of Specific Materials

(See also Revs. 580, 581, 584, 600)

635. C. Sykes, "Steels for use at elevated temperatures," *J. Iron Steel Inst.*, July 1947, vol. 156, pp. 321–338.

The paper surveys recent experimental work conducted by the late W. H. Hatfield on a number of alloyed steels subject to various types of heat-treatment for use at service temperatures up to 800 C in the form of large forgings, and compares the creep performance of these steels with the performance of carbon and carbon-molybdenum steels. The evaluation of creep performance is based on the "Hatfield time-yield stress," that is, the stress which produces an average creep rate of 0.0001 per cent per hour between the 24th and 72nd hour of the creep test, and on the stress producing creep-rupture after 1000 hr.

The test results show that the most satisfactory creep performance of ferritic steels at temperatures up to 600 C was that of molybdenum-vanadium steel, and that nickel-chromium-molybdenum steels and chromium-molybdenum steels were superior to carbon-molybdenum steel only at temperatures below 450 C. The creep performance of each type of steel is a function of its heat treatment; the best results were achieved by air-cooling from around 1000 C.

While the ferritic steels are usually limited to short-term applications in the range of 450 to 600 C unless very low stresses can be used, austenitic steels containing chromium in excess of 12 per cent are used for higher temperatures. The creep resistance of such steels which also contain nickel, molybdenum, cobalt and either titanium or columbium is quite exceptional, particularly after the "triple treatment" which involves air-cooling from three different temperatures.

An extensive bibliography on creep- and heat-resisting steels covering the period 1937 to 1947 is included.

A. M. Freudenthal, USA

636. J. R. M. Radok, J. P. O. Silberstein, and H. A. Wills, "A new theory for the strength of wooden box beams," *Counc. sci. indust. Res. Div. Aero. Rep.*, no. ACA-40, May 1948, pp. 1–37.

A new hypothesis is advanced to account for the fact that the modulus of rupture of a wooden beam is greater than the ultimate compressive stress of the wood as determined by a simple compression test. For the stress distribution at failure it is assumed that the stress on the compression side increases from a value f_p in the extreme fibers to a value f_u at some distance below them, and then decreases. To simplify the analysis, the variation in stress is taken to be linear in both regions.

It is further assumed that f_u is greater than the ultimate compressive stress f_c given by a simple compression test. This assumption is based on the variability of the compressive strength in a piece of wood. In a compression test, the strength will tend to be determined by that of the weaker regions. In a beam, failure will start somewhere along the extreme fibers in a region of low compressive strength. It is to be expected that below this region there will be a region of higher compressive strength.

The problem is then considered to be a statistical one of determining from test data the broken line representing the stress distribution at failure in terms of quantities defining the geometry of the section. An analysis was made of an extensive series of tests of box beams of two species of wood, all of the same width and depth but with various thicknesses of flanges. Curves are given showing the "form factor" as a function of appropriate geometrical parameters. A supplementary series of tests was made on beams that were solid except for a longitudinal slit placed at various depths and extending between the two sections at which the loads were applied. Information from this series of tests is used in discussing various hypotheses that have been or may be advanced to explain the phenomenon under consideration.

H. W. March, USA

637. R. L'Hermite, and G. Tournon, "Vibration of fresh concrete (La vibration du béton frais)," *Bâtim. Trav. publics*, Feb. 1948, no. 11, pp. 3–75.

In this article, the authors divide their responsibility for the subject matter, the senior author giving the purely theoretical discussion of the effect of vibrating fresh concrete while the junior author presents a discussion of the experimental results.

The treatise is built around the premise that any study of the effect of vibration on fresh concrete should be concerned with its properties before it has hardened. In this connection the concept of an internal friction between the particles of the concrete, combining dry or Coulomb friction with viscous friction, is introduced. An analysis is developed to show how this apparent internal friction is decreased as a result of the "agitation pressure" set up by the vibration. This agitation pressure is proportional to the percentage of voids as well as to the external vibration. The instantaneous viscosity enters into the compaction of the concrete as an exponential function.

In the experimental discussion the inadequacies of the slump and flow tests (ASTM) are reviewed and a workability test proposed by Faury and a "remolding" test proposed by B. Bährner are discussed. For the present investigation, however, a torsional shear test is proposed. The torsional apparatus and testing procedure used in these tests are then described. Data on tests of dry elements of concrete (cement, sand, and gravel) are included as well as those for the fresh concrete. The effect of water content and gradation of aggregate on the coefficient of viscosity were also investigated. The segregation in the freshly vibrated concrete due to the vibration was measured by an ingenious method dependent upon the location of the center of gravity of the mass. The segregation together with the percentage of voids were used as the prime indexes of the quality of the vibrated concrete. These factors were correlated with the apparent coefficient of internal friction and also with the compressive strength.

Frank J. Mehringer, USA

638. G. Bickley Remmey, "Upper useful limits of commercial superrefractories," *Bull. Amer. ceram. Soc.*, Dec. 1948, vol. 27, pp. 447–485.

Tests are described for the determination of the safe upper temperature limits of eighteen brands of fabricated mullite and alumina refractories and one sample each of zircon and zirconia. Horizontal and vertical changes were measured after reheat tests at temperatures in the range 3000 to 3500 F over a period of four hours, and were used to assess the shrinkage effects. Pyrometric Cone Equivalent tests and load tests at a pressure of 25 psi at 2950 F were also made. The temperatures were measured by a platinum-platinum rhodium couple and an optical pyrometer.

It was considered that a useful criterion was a horizontal shrinkage of less than 1.5 per cent together with a maximum increase in vertical shrinkage over the horizontal of 0.2 per cent, and that a refractory having these properties would be suitable for the interior of a furnace with all sides exposed to the heat in the absence of slag attack or excessive conditions of stress. On this

basis, improvements in the new mullite and alumina refractories extend the upper limit of use by some 225 to 325 F and 200 to 500 F respectively. In the mullite refractories, the improvement is due to the presence of interlocking crystals which do not soften much below their dissociation temperature of 3326 F. The alumina refractories have been improved by the elimination of components which form fluid glasses at the higher temperatures. The reactions of zircon and zirconia with mullite and alumina are also discussed. B. W. Mott, England

639. **H. S. Kalish and F. J. Dunkerley, "The low temperature properties of tin and tin-lead alloys,"** *Metals Technol.*, Sept. 1948, vol. 15, T.P. no. 2442, 20 pp.

Very pure tin-lead alloys containing up to 50 per cent lead were cast into tension bars in a steel mold. The tensile strength, elongation and reduction of area were determined in the range from +27 C to −196 C after annealing 200 hr at 175 C.

Pure tin shows a sharp transition temperature at approximately −140 C, below which the reduction of area goes down sharply and the type of fracture changes from shear to cleavage. The tensile strength reaches a maximum (14 psi) in this range and then also sharply decreases. At lead contents up to 1.66 per cent (the saturation limit at 175 C) the maximum tensile strength is little effected, but the transition temperature is raised to −60 C; the tensile strength increases in the ductile range and the ductility decreases. At lead contents up to 10 per cent the maximum tensile strength strongly increases and then diminishes slightly, while the transition temperature continuously goes down (21 psi for 10 per cent lead at −120 C; 19 psi for 50 per cent lead at −160 C), resulting in a considerable improvement of the strength and ductility at low temperatures. Alloys containing more than 70 per cent lead probably remain ductile down to temperatures near absolute zero. J. H. Palm, Holland

640. **G. Malmlöw, "Thermal ageing properties of cellulose insulation materials"** (in English), *Trans. roy. Inst. Technol. Stockh.*, 1948, no. 19, pp. 9–67.

The paper studies the behavior of insulating materials in actual service by investigating isolated material under conditions designed to simulate the influencing factors. The investigation is limited to mechanical properties of cellulose electrical insulation, such as tensile, bursting, tearing and folding strengths and elongation. A compilation of earlier investigations (Clark, and others) and extensive references are given. The results are compared with Montsinger's eight-degree rule. The most probable relative deterioration rates for temperatures between 90 and 140 C are tabulated. Ageing curves are given and represented by the sum of two exponential functions, both with temperature-dependent time constants. The responsible factor is definitely shown to be the thermal decomposition of the cellulose. Impregnation in oil does not significantly lower the rate of decomposition, but the effect of oxygen above the oil or the presence of water is shown to affect decomposition. Ellis F. King, USA

641. **William M. Roberts and George J. Heimerl, "Elevated-temperature compressive stress-strain data for 24S-T3 aluminum-alloy sheet and comparisons with extruded 75S-T6 aluminum alloy,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1837, Mar. 1949, pp. 1–11.

Compressive stress-strain tests were made on 24S-T3 aluminum-alloy sheet at temperatures up to 700 F, and the results compared with similar data for extruded 75S-T6 aluminum alloy. Compressive yield strength and modulus for the 24S-T3 sheet were found to decrease as expected with increase in temperature, but the percentage decrease appears to be appreciably less than for 75S-T6 extrusions. John E. Goldberg, USA

642. **L. Northcott and D. McLean, "Some properties of titanium steels,"** *J. Iron Steel Inst.*, Dec. 1947, vol. 157, pp. 492–512.

This paper describes an investigation into the effect of up to 6 per cent of titanium on the general mechanical properties of plain carbon and low-alloy steels. The carbon steels contained from 0.1 to 1.0 per cent carbon. The low-alloy steels were chromium-molybdenum, manganese-molybdenum, chromium, and nickel steels. For the plain carbon steels the investigations include: the effect of titanium on the tensile strength, hardness, and impact strength of the annealed materials; the influence of titanium and atmosphere (air or vacuum) on the hardness after quenching; the manner in which titanium affects the hardness after tempering and the quench-aging of the steels; the effect of titanium upon the ultimate strength at elevated temperatures; and the effect of titanium upon the hardenability as determined by the Jominy test. Similar but less extensive tests were made on the low-alloy steels.

The authors conclude that if the steel is quenched from a high temperature (1235 C) and if the ratio of titanium to carbon is 4 to 1 by weight, high strength and hardness after quenching will be maintained up to 600 C. This effect is accentuated by the addition of 1 per cent of chromium. They also found that if the steel contains at least 4 per cent of titanium, precipitation-hardening by Fe_2Ti occurs at temperatures of 500 to 700 C. The rate of hardening is increased by the addition of 3 per cent of nickel. They further found that titanium increases the hardenability (Jominy test) to about the same extent as chromium.
Evan A. Davis, USA

643. **R. L'Hermite, "Shrinkage of cements, mortars and concretes (Le retrait des ciments, mortiers et bétons),"** *Bâtim. Trav. publics*, June 1947, no. 37, pp. 2–79.

This paper presents a comprehensive study of the shrinkage of cements, mortars and concrete. It gives a review of current work done throughout Europe and America as well as the results of the studies made by the author. The work is divided into two parts. Part I deals with the experimental results of numerous researches in the field and is also a comprehensive study of numerous means and methods of measuring the shrinkage. The effect of such variables as type of cement, proportions of mix, admixtures, and curing history are discussed in detail.

Part II deals with the theory of shrinkage in neat pastes, mortars and concrete. The drying-out of the material, the primary cause of shrinkage, when treated analytically results in a differential equation of the same form as that of Fourier for the heat transfer through a body. Semiempirical relations are necessary, however, to make the theoretical results agree with the experimental relations. A bibliography is included.
Frank J. Mehringer, USA

644. **W. C. Lewis, "Fatigue of sandwich constructions for aircraft,"** *For. Prod. Lab. Rep.*, no. 1559, Dec. 1946, pp. 1–11.

Results are reported for fatigue tests made in shear on cellular cellulose acetate. An S-N curve was obtained for one condition of loading, namely, for a ratio of minimum to maximum stress of 0.1. For this condition of loading, the tests indicate an endurance limit of approximately 37 per cent of the static strength.

This report is the first of a series intended to compare the be-

havior of different core materials and sandwich constructions subjected to repeated loads. Frank Baron, USA

645. J. Neill Greenwood and J. H. Cole, "The influence of various factors on the creep of lead alloys," *Metallurgia*, Jan. 1949, vol. 39, pp. 121–126.

Data are presented from the results of long-time tests (up to nine years in some cases) to study (*a*) the influence of steady stresses at 20 C on lead alloys containing copper or silver; (*b*) influence of steady stress on similar specimens at 50 C; (*c*) influence of steady stress with vibration and the effect of previous heat-treatment on lead alloys containing 0.075 per cent copper or 0.03 per cent silver.

At ordinary temperature the addition of 0.06 per cent Cu to pure lead decreased the creep rate for a stress of 350 psi by a factor of about ten, but this advantage was lost completely if the Cu was coarsely dispersed. At 50 C this factor was reduced to about four; as the Cu content increased the alloys appeared to become more sensitive to temperature increase. The creep rate was markedly increased by the presence of superimposed vibration. The silver leads tended to creep at a constant rate for six years under a given load. In general, the addition of silver did not appear to promote increased resistance to creep of lead consistently, but did result in increasing the elongation prior to fracture. Recrystallization occurring during a test coarsened the grain and markedly lowered the subsequent creep rate; hence tabular results of total extension for a given creep time may often be misleading if extrapolated.

Comparison of the results indicated good agreement with those of American investigators [C. W. Dollins, *Univ. Ill. engng. Exp. Sta. Bull.*, no. 378, July 1948; and G. R. Gohn and W. C. Ellis, *Amer. Soc. Test. Mat. Proc.*, 1948]. T. J. Dolan, USA

646. William C. Stewart and W. Lee Williams, "Effects of inclusions on the endurance properties of steels," *J. Amer. Soc. nav. Engrs.*, Nov. 1948, vol. 60, pp. 475–504.

An experimental study was made of the effect of nonmetallic inclusions on the acceptability of steel forgings. A few fatigue tests were performed on samples from defective parts and certain theories regarding stress concentrations were confirmed by experiments.

The authors note that inclusions are most serious near the surface where stresses are highest and also near changes of sections, where stress concentrations already exist. Inclusions are most serious when they occur in hard steels; however, large forgings are usually made of relatively soft steels. Stress concentrations vary markedly with the orientation of an inclusion. Long narrow inclusions in the direction of the stress have little effect; perpendicular to the direction of the stress their effect is marked. Globular inclusions with no special orientation have little effect on endurance strengths. Small inclusions are less detrimental than large ones. As the shaft size decreases, the stress-concentration factor increases.

No rigid specifications can be written at present on the acceptable limits of nonmetallics in large forged parts. Doubtful cases should be considered individually by experienced inspectors.

Andrew Brodsky, USA

647. C. B. Norris, Fred Werren, and P. F. McKinnon, "The effect of veneer thickness and grain direction on the shear strength of plywood," *For. Prod. Lab. Rep.*, no. 1801, July 1948, pp. 1–40.

The effects of veneer thickness and grain direction were investigated by panel-shear, block-shear and tension-shear tests of plywood samples made from Douglas fir, khaya, yellow birch, sweet gum and yellow poplar. Three constructions of plywood with veneer thicknesses of $^1/_8$, $^1/_{16}$, $^1/_{32}$ and $^1/_{64}$ in. were used.

An empirical relation is given expressing the shear strength of a plywood panel in terms of number of plies, thickness of plywood and shear strength of the solid wood of the species used in the plies. Constants in this expression are determined by the kind of glue and gluing technique employed, and the depth of checks in the veneers. The relation appears to be reasonably well-confirmed by the tests.

Comparison of the three types of shear tests is also made. The block, panel and tension-shear tests gave strength values in that order from high to low. The results given are average ultimate strengths; buckling did not enter into the investigation.

Henry A. Lepper, Jr., USA

648. J. O. Everhart, "Behavior of fired-clay ring sections under load," *J. Amer. ceram. Soc.*, Feb. 1949, vol. 32, pp. 53–65.

An accurate formula is derived for calculating the stresses and strength of a circular ring section of a brittle elastic material in terms of the dimensions and strength properties, and a discussion is given of its application to the problem of the strength of sewer pipes and drain tiles in the light of earlier work which had shown that clay products deform elastically and obey Hooke's law up to failure. Experiments were carried out in which the deformation of a vitrified-clay sewer pipe under load was measured at various positions by means of electric strain gages placed at right angles to the axis of the pipe.

The relationship between the deformations found on the inside and outside of the pipe was in full agreement with the derived formula, and showed that in the absence of any abnormal conditions failure of sewer pipes will always be initiated on the top inside surface, which is the position of maximum strain. Knowing the value of E for a given material, accurate values for the stresses at the top inside and outside can be calculated from the measured strains. In addition, the equation can be used for evaluating the relative effect of adding thickness to the inside and outside surfaces of a pipe and for a comparison of the strengths of pipes of different diameters.

Examples are given to show how the results of this work can be applied to problems involving the changes in pipe section required to bring about a desired increase in strength.

B. W. Mott, England

649. K. W. Hillier and H. Kolsky, "An investigation of the dynamic elastic properties of some high polymers," *Proc. phys. Soc. Lond.*, Sect. B, Feb. 1949, vol. 62, pp. 111–121.

The authors describe an apparatus for the investigation of the transmission of sound along filaments at frequencies between 1000 and 6000 cps, both for unstrained specimens and for specimens which are being elongated at a constant rate of increase of strain.

In the unstretched specimens the dynamic value of Young's modulus obtained from the measurements of the velocity of sound was found to be several times as great as the limiting gradient of the stress-strain diagram. The values of the dynamic moduli for both polythene and nylon, as the filaments were stretched, first showed a slight decrease followed by a rapid rise. In the case of neoprene, however, the dynamic modulus increased steadily.

On the other hand, the gradients of the stress-strain curves for polythene and nylon decreased continually with strain, showing that the effects of plastic flow completely masked the changes in elastic behavior. For neoprene, the gradient increased but at a

much slower rate than the dynamic modulus. The latter had increased more than a hundredfold when the specimen had reached 5.8 times its original length.

The results of polythene at 1500, 3000 and 6000 cps showed that the damping increases with the frequency, while there is comparatively little rise in the velocity of sound, and hence in the dynamic modulus.

The damping coefficient of unstretched neoprene was found to be very much higher than the damping coefficients obtained for polythene and nylon. Up to a strain of 0.8 the value of the damping coefficient showed little change; for strains greater than this, however, it decreased rapidly and had reached about one eighth of its original value at a strain of 2.0. George Halsey, USA

Mechanics of Forming and Cutting

(See also Rev. 621)

650. **R. Hill, "The calculation of stresses in the ironing of metal cups,"** *J. Iron Steel Inst.*, Jan. 1949, vol. 161, pp. 41–44.

The paper describes a method for estimating the force required to draw a thin-walled cup by forcing it through a closely fitting circular die with a conical working surface. Although the author refers to an approximate method described in several papers by George Sachs and his collaborators, he does not mention that other investigators in the United States and in Germany have given thought to the mechanical processes of drawing of cylindrical cups, cartridges of brass and steel, thin-walled tubing, etc., by analyzing the plastic distortion of such bodies during their forming.

Previous methods are characterized by the author as "approximate because they assume homogeneous states of stress in the working portion of the thin-walled cylinder satisfying a condition of plasticity." In contrast he states that his own analysis, which is based on a determination of the nonhomogeneous stresses in a wedge-shaped region in which the elements of the body are plastically deformed, would furnish a more satisfactory determination of the drawing force. This may have to be proved in more detail because the author's analysis of the distribution of stresses in the wedge-shaped working region also had to be based on a number of idealizations regarding the true conditions encountered in those parts of the body which are deformed under local highly concentrated lateral pressures (neglect of friction between cup and die, of the influence of the adjacent portions of the body which do not participate in visually perceptible distortions, and other factors). The author assumes, for example, that the two boundaries of the plastic zone coincide with two lines of slip which, on the side of the punch, terminate at the same point; neither of these assumptions is very convincing.

The details of the analysis of the distribution of stress in the wedge-shaped plastic region are not given in the present paper, but in a preceding one in which a theory of wire drawing was proposed by Hill and Tupper [*J. Iron Steel Inst.*, 1948, p. 177]. Some reference to the influence of friction between cup and punch and of strain hardening is made. The drawing force is plotted as a function of the reduction, for various angles of the conical die.

A. Nádái, USA

Hydraulics; Cavitation; Transport

651. **M. E. Von Seggern, "Integrating the equation of non-uniform flow,"** *Proc. Amer. Soc. civ. Engrs.*, Jan. 1949, vol. 75, pp. 105–122.

In addition to the "hydraulic exponent n" of Bakhmeteff [*Hydraulics of Open Channels*, McGraw-Hill, 1932] the author introduces a secondary exponent m, defined by means of the equation

$$(M_c/M)^2 = (y_c/y)^m$$

where

$$M = (A^3/b_w)^{1/2},$$

A being the cross-sectional area, b_w the width of water surface, y_c the critical depth, and M_c the value of M for y_c.

He gives numerical tables of two functions introduced by integration, charts for determing m and n, and examples of practical computations. A. Craya, France

652. **Alexander H. Jameson, "Flow over sharp-edged weirs: effect of thickness of crest,"** *J. Instn. civ. Engrs.*, Nov. 1948, vol. 31, pp. 36–55.

This is a very interesting analysis of the experimental results obtained by Bazin, Fteley and Stearns, and Shimoyama on sharp-edged weirs without end contractions. For the first time an attempt is made to take into account the influence of the thickness of the crest. The motion depends evidently upon the three ratios h/b, h/p, h/t (h is the head over the weir, b the breadth of the weir, p the height of the weir above upstream floor, t the thickness of crest), and in addition upon the Reynolds number r and the Weber number W (h being always the linear dimension).

The author shows that the discharge coefficient of the usual formula can be expressed, for the two-dimensional motion, as a function of two numbers only, the products $(R) = R(h/p)^6$ and $(W) = W(h/t)^3$. As (W) becomes greater than about 11.6, it does not any more affect the value of the discharge coefficient. A comparison is made between the author's formulas and those of Bazin and Rehbock. Correction coefficients are also given for three-dimensional motion, that is, for values of h/b greater than about 0.2. Duilio Citrini, Italy

Incompressible Flow: Laminar; Viscous

(See also Revs. 608, 651, 665, 666, 668, 669, 691)

653. **E. Fage and M. Vernet-Lozet, "A method of calculation of velocities at the surface of an airfoil in an ideal incompressible fluid (Une méthode de calcul des vitesses à la surface d'un profil d'aile, en écoulement plan d'un fluide parfait incompressible),"** *C. R. Acad. Sci. Paris*, Dec. 20, 1948, vol. 227, pp. 1339–1341.

The author points out that if the flow about an airfoil (in the complex ζ-plane) is transformed into the flow about a circle (in the z-plane), the ratio of the complex velocities at corresponding points is $d\zeta/dz$, and the speed ratio is $|d\zeta/dz|$. Since $\log(d\zeta/dz)$ is an analytic function, it follows that $\log|d\zeta/dz|$ can be computed from the Poisson integral relation if the argument of $d\zeta/dz$ is known. The author suggests that the argument of $d\zeta/dz$ for a given profile may be estimated from that for a theoretically known profile by correcting this argument by the angular difference of the slopes of the profiles at corresponding chord points.

H. J. Stewart, USA

654. **M. L. Albertson, Y. B. Dai, R. A. Jensen, and Hunter Rouse, "Diffusion of submerged jets,"** *Proc. Amer. Soc. civ. Engrs.*, Dec. 1948, vol. 74, pp. 1571–1596.

The authors study the problem of turbulent diffusion of submerged jets, searching for the laws according to which the speeds vary within the jet and the surrounding region. This research is not based on any particular theory of turbulence (as the theory of the "momentum transport" or the "vorticity transport"), in

contrast to other work on the same subject. It rests only on the following hypotheses: (*a*) the pressure is hydrostatically distributed throughout the flow; (*b*) the diffusion process is dynamically similar under all conditions; (*c*) the longitudinal component of velocity within the diffusion region varies according to the normal probability function at each cross section.

These assumptions lead to expressions for the volume, momentum and energy flux through any transversal section of the submerged jet, and define the distribution of longitudinal and lateral velocity. A single experimental constant is introduced for each of the two cases of a jet emerging from a slot and from a circular orifice. The experimental results of the authors agree with those calculated, except for the results relating to the laws of distribution of transversal velocity. Carlo Ferrari, Italy

655. Caïus Jacob, "On the extension of certain integral formulas to the flow of perfect fluids (Sur l'extension de certaines formules intégrales aux écoulements des fluides perfaits)," *C. R. Acad. Sci. Paris*, May 31, 1948, vol. 226, pp. 1793–1795.

Levi-Civita [*Atti Ist. Veneto Sci. Lett. Arti.*, 1905, vol. 64, pp. 1465–1472] generalized a formula of Blasius, useful in calculating the force on a body in incompressible irrotational flow, for the three-dimensional case, as follows:

$$\frac{1}{2}\int_S v^2\mathbf{n}d\sigma = \int_S (\mathbf{n}\cdot\mathbf{v})\mathbf{v}d\sigma \qquad (1)$$

where $\mathbf{v}$ is the fluid velocity vector, S is a closed surface including no singularities and $\mathbf{n}$ is the unit inward normal to S. The second Blasius formula was generalized by von Mises [*Bull. Amer. math. Soc.*, 1944, vol. 50, pp. 599–611], who obtained:

$$\frac{1}{2}\int_S v^2(\mathbf{r}\times\mathbf{n})d\sigma = \int_S (\mathbf{v}\cdot\mathbf{n})(\mathbf{r}\times\mathbf{v})d\sigma \qquad (2)$$

where $\mathbf{r}$ is the position vector. It is shown that (2) can be obtained directly from Green's formula. Another extension by von Mises, of a formula of Cauchy, is also discussed.

It is shown that there are formulas analogous to (1) and (2) for the steady irrotational flow of a barotropic fluid:

$$\frac{1}{2}\int_S K(v^2)\mathbf{n}d\sigma = \int_S \rho(\mathbf{n}\cdot\mathbf{v})\mathbf{v}d\sigma,$$

$$\frac{1}{2}\int_S K(v^2)(\mathbf{r}\times\mathbf{n})d\sigma = \int_S \rho(\mathbf{n}\cdot\mathbf{v})(\mathbf{r}\times\mathbf{v})d\sigma,$$

where ρ is the density and $-K(v^2)$ denotes $-\int \rho dv^2$, which is, by the equation of motion, the pressure.

Courtesy of Mathematical Reviews W. R. Sears, USA

656. F. R. Eirich and D. Tabor, "Collisions through liquid films," *Proc. Camb. phil. Soc.*, Oct. 1948, vol. 44, pp. 566–580.

Approximate analysis of the hydrodynamic behavior of a liquid layer interposed between two colliding solid surfaces is given. The model considered is that used in typical fall-hammer experiments in investigations of the impact sensitivity of explosives. A flat circular hammer falls from a specific height to a flat anvil covered with a thin film of liquid. The pressure distribution, velocity of flow, and rate of shear in the liquid film are calculated. Estimates are also made of the temperature rise occurring in the liquid film as a result of viscous flow and as a result of the adiabatic compression of gas bubbles that may be trapped in the liquid film. The analysis assumes that elastic or plastic deformation does not occur at the metal surfaces. The equations developed are applied in typical fall-hammer experiments in the impact sensitivity of nitroglycerine.

Stuart R. Brinkley, Jr., USA

Compressible Flow, Gas Dynamics

(See also Revs. 571, 608, 655, 683)

657. Howard W. Emmons, "Gas dynamics tables for air," Dover Publications, New York, 1947. Paper, 9.25 × 6.1 in., 46 pp., 3 figs., $1.75.

A brief sketch of the nomenclature and computation formulas for various gas-dynamical (nondimensional) quantities is given. Shock conditions and computations of flows around a corner are also explained. The following four tables are computed:

1 Isentropic gas-dynamical functions for the Mach number M in the range 0 to 25 (in 0.001 steps from $M = 0$ to 0.5; in 0.01 steps from $M = 0.5$ to 0.8; in 0.001 steps from $M = 0.8$ to 1.2; in 0.01 steps from $M = 1.2$ to 5; and in steps of 1 from $M = 5$ to 25).

2 Gas-dynamical functions for normal shocks with $M = 1$ to 25 (in 0.01 steps from $M = 1$ to 2; in 0.1 steps from $M = 2$ to 25).

3 Characteristics table for "proper angle" ν and Mach angle α and Mach number M for the range of $\nu = 0$ to 130.5 in 0.5 steps.

4 Acoustic velocity versus temperature table in the range from 300 R to 698 R in steps of 2.

The tabulated functions are also presented graphically.

S. S. Shu, USA

658. G. N. Ward, "The pressure distribution on some flat laminar aerofoils at incidence at supersonic speeds," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2206, Mar. 1946 (issued in 1948), pp. 1–11.

Under the assumptions of linearized compressible-flow theory, angle-of-attack load distributions for particular polygonal plan forms in supersonic flow are shown to result from the superposition of load distributions over certain semi-infinite wings. Solutions for these latter cases are obtained after first expressing the general solution of the hyperbolic partial differential equation as a contour integral and then transforming to conical flow variables. Explicit solutions are given for tip regions of swept wings, the center region of a sweptback wing with supersonic leading edges, a triangular wing inside the Mach cone from the vertex, and a flat vane at incidence in a free jet. Max A. Heaslet, USA

659. W. Frössel, "Condensation shock in compressible flow over curved walls (Kondensationsstösse bei kompressibler Strömung an gewölbten Wänden)," *Z. angew. Math. Mech.*, May 1947, vol. 25-27, pp. 50–53.

Experimental data are given showing the position of condensation shocks in channels in which one wall is plane and the other is a circular cylindrical surface. The positions of the condensation shock (found by schlieren methods) and the pressure distributions are given for four Mach numbers near 6 and two channels with different throats. Only the case where the upstream atmosphere has a relative humidity of 77 per cent is considered.

Harry E. Moses, USA

660. H. R. Ivey, E. B. Klunker, and E. N. Bowen, "A method for determining the aerodynamic characteristics of two- and three-dimensional shapes at hypersonic speeds," *Nat. adv. Comm. Aero. tech. Note*, no. 1613, July 1948, pp. 1–21.

The problem of determining the aerodynamic characteristics of

two-dimensional airfoils of bodies of revolution at hypersonic speeds (that is, for Mach numbers of the order 30 to 40), in air sufficiently dense to be considered a continuum, becomes simplified for the reason that the shock wave becomes essentially in line with the surface. The flow direction changes abruptly at the surface from the free-stream inclination to the local inclination of the surface. The authors analyze the conditions that must exist in this boundary layer of high-pressure air on the assumption of a specific heat ratio of unity, which is reasonable in this case. The results are in general agreement with those obtained earlier by Busemann. The flow is assumed to leave the wall at the point where the pressure falls to zero. Lift and drag coefficients are computed for a number of simple shapes.

Stewart Way, USA

661. **F. Pfeiffer and W. Meyer-König, "The characteristics method applied to stationary two-dimensional and rotationally symmetrical gas flows,"** *Nat. adv. Comm. Aero. tech. Memo.*, no. 1211, Mar. 1949, pp. 1–20 (transl. from *Zent. wiss. Berichtswesen Berlin Forschungsber.*, no. 1581, Mar. 20, 1942).

The mathematical theory of characteristics is used to derive a numerical method for computing two-dimensional and axially symmetric supersonic flows. Hideo Yoshihara, USA

662. **B. L. Hicks, "On the characterization of fields of diabatic flow,"** *Quart. appl. Math.*, Jan. 1949, vol. 6, pp. 405–416.

In this paper, the author continues some of his earlier studies of manipulation of the equations of flows with heat addition. He studies the equations for a vector field codirectional with the velocity field, particular attention being paid to the appearance of the equations for various choices of the scalar factor of proportionality. The consequences of assuming that the vector field is irrotational are investigated. The partial differential equation of the potential function is established and consideration is given to the conditions under which it is elliptic, parabolic or hyperbolic. The paper concludes by examining a number of special flows for which the determinant of the coefficients of the highest-order terms in the partial differential equation is constant.

Francis H. Clauser, USA

663. **E. A. Krasilshchikova, "Influence of the vortex filament in the steady motion of a wing at supersonic velocity"** (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Dec. 21, 1947, vol. 58, no. 6, pp. 989–991.

In *Doklady Akad. Nauk SSSR*, vol. 58, no. 4, the author developed formulas for the "linearized" upwash in the neighborhood of subsonic leading edge of a flat wing of arbitrary plan form in a supersonic stream. The present note summarizes the extended application of the method to the upwash in the neighborhood of a subsonic trailing edge and to the problem of a wing of very small aspect ratio. The basic unqualified assumption is made that the slope of the "linearized" streamlines is continuous at the subsonic trailing edge and at the Mach cone issuing from the tip of the wing.

The use of oblique coordinates along Mach lines in the plane of the wing and of the source solution leads to several forms of Abel's integral equation. The final result for the slope $\vartheta(x, y)$ of the streamline in the region affected by the trailing edge should reduce to Evvard's results [*Nat. adv. Comm. Aero. tech. Note*, no. 1585, May 1948, Eqs. 22a and 23a] after the continuity conditions mentioned are used. No discussion of the general problem of the trailing-edge conditions and the equivalence with the Kutta-Joukovski condition is included. M. V. Morkovin, USA

664. **M. J. Lighthill, "The flow behind a stationary shock,"** *Phil. Mag.*, Feb. 1949, vol. 40, pp. 214–220.

The steady inviscid adiabatic flow of a gas is shown to satisfy the linearized potential equation provided the flow is approximately uniform. The gradient of the potential function is given by $\nabla\phi = (u', v, w)$ where $\rho_1 a_1 u' = p_1 - p$, and v and w are the usual perturbation velocity components.

This equation is used for studying the two-dimensional flow produced by the passage of a uniform stream through a shock of nearly uniform (but not small) strength. Calculations are made which show that small disturbances from an airfoil reflect from the shock wave along characteristics with a strength dependent on the initial Mach number and the shock strength. For very large Mach numbers the disturbances are reflected with very little loss in strength.

It is also shown that the theory can be applied to the study of thin supersonic airfoils with slightly rounded leading edges. Outside of the region near the leading edge the flow will be approximately uniform, so that the ordinary theory of thin supersonic airfoils can be applied using u', v instead of u, v. It is concluded that the leading-edge bluntness does not affect the surface pressures except locally. Lester L. Cronvich, USA

665. **A. R. Kuhlthau, "Air friction on rapidly moving surfaces,"** *J. appl. Phys.*, Feb. 1949, vol. 20, pp. 217–223.

This paper describes an attempt to measure the gas friction on a surface most of which is moving at a prescribed speed. The range of prescribed speeds was 400 to 1700 ft per sec and the pressure ranged from 5 to 200 microns. The friction was generated on a cylinder rotated about its axis by means of an air-driven turbine and picked up in a torsion wire which supported a floating cylindrical shell surrounding the rotor. The relationship between torque and pressure at constant speed gave a family of curves indicative of ordinary viscosity.

Calculated torques in the "free molecular" range were high relative to the measured experimental values. The reviewer believes this disagreement was due in large part to the fact that the speed imparted to the entrained gas was not taken into account. End effects should also have been carefully explored, perhaps by using longer hollow rotors which are dynamically balanced to assure stable motion. M. G. Scherberg, USA

Turbulence, Boundary Layer, etc.

(See also Rev. 654)

666. **M. Tournier and M. Bassière, "On a solution of the boundary layer equations (Sur une solution des équations de la couche limite),"** *C. R. Acad. Sci. Paris*, Apr. 5, 1948, vol. 226, pp. 1124–1126.

In this note the coefficient of friction and boundary-layer thickness are treated for infinite and finite plates suddenly set in uniform motion from a stationary condition in a viscous fluid. The solutions obtained vary less than 2 per cent from the Blasius values for steady motion. The analytic approximation selected for the velocity distribution proved to be more useful than the usual series approximation. Richard G. Folsom, USA

667. **Jean Bass, "On the mathematical basis of Heisenberg's turbulence theory (Sur les bases mathématiques de la théorie de la turbulence d'Heisenberg),"** *C. R. Acad. Sci. Paris*, Jan. 17, 1949, vol. 228, pp. 228–229.

The author notes that in a recent important paper [*Z. Phys.*, 1948, vol. 124, p. 628], Heisenberg makes use of a Fourier series

development of the velocity distribution in a field of homogeneous turbulence. This is not rigorous, and so the author substitutes Fourier-Stieltjes stochastic integrals. None of the results of Heisenberg's paper is affected. The author gives, in passing, the exact solution of Heisenberg's equation for the equilibrium spectrum. G. K. Batchelor, England

668. H. Görtler, "Calculation of laminar boundary layers by the method of finite differences (Ein Differenzenverfahren zur Berechnung laminarer Grenzschichten)," *Ingen.-Arch.*, 1948, vol. 16, no. 3/4, pp. 173–187.

The object of this paper is to develop a numerical method for calculating the laminar boundary layer in two-dimensional incompressible flow which would produce solutions of useful accuracy without iteration. The method given is based on approximating the partial derivatives by difference quotients formed with $u_{k+1,i} - u_{k-1,i}$ rather than with $u_{k,i} - u_{k-1,i}$.

As an example, the steady flow over a flat plate is computed and compared with numerical solutions computed by S. Haworth [*Proc. roy. Soc. Lond. Ser. A*, 1938, vol. 164, p. 547] from the Blasius solution. Taking the latter as exact, the error in the velocity profile at any station (relative to the associated main-stream velocity) did not exceed one per cent. For each profile it took about an hour to compute the ten to fifteen points required to obtain this accuracy. The boundary layer in the case of linearly decreasing main-stream velocity is also calculated and compared with known solutions. It is indicated how the method can be extended to the case of axially symmetric flow, to the case with blowing and suction, and to the case of unsteady flow.

J. S. Isenberg, USA

669. H. Görtler, "On the approximation of stationary laminar boundary flow by means of the truncated Blasius series (Zur Approximation stationärer laminarer Grenzschichtströmungen mit Hilfe der abgebrochenen Blasiusschen Reihe)," *Arch. Math.*, 1948/49, vol. 1, no. 3, pp. 235–240.

The author discusses the truncated Blasius series for the stream function in the computation of the boundary layer on a cylinder, from the stagnation point to the separation point. He establishes two criteria for the choice of the number of terms which may be applied before the boundary layer is actually computed. He compares the results of taking five, seven, or nine terms for the case of the circular cylinder with the result of a difference method, and with the experimental results of Hiemenz.

John A. Lewis, USA

670. C. W. Jones, "On a solution of the laminar boundary-layer equation near a position of separation," *Quart. J. Mech. appl. Math.*, Dec. 1948, vol. 1, pp. 385–407.

The purpose of this paper is to make a more detailed investigation by computational methods of a solution by S. Goldstein of the laminar boundary-layer equations in the upstream neighborhood of a separation point [S. Goldstein, "On the laminar boundary-layer flow near a position of separation," *Quart. J. Mech. appl. Math.*, 1948, vol. 1, pp. 43–69; see Rev. **1670,** Nov. 1948]. In particular, in Goldstein's paper it was found necessary to make several conjectures which he was not able to verify at that time. One of these was the vanishing of a certain integral. In the present paper this is verified within the accuracy of the computations. In the course of this computation tables are prepared of the functions f_0, f_0', f_1, f_1', f_2, f_2', f_3, f_3' and of two functions related to f_4 and f_5 (see afore-mentioned review of Goldstein's paper for the meaning of these functions).

A second conjecture was that the solution is uniquely determined when a certain constant has been determined and that this constant may be determined by a suggested method. This is also verified, in so far as numerical computation can do so, by comparison with computations by Hartree, which were carried out by starting at the stagnation point instead of the separation point.

J. V. Wehausen, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 608, 658, 663, 676, 679)

671. M. Panetti, "On the vertical flight of the helicopter (Sul volo verticale delle elicottero)," *Aerotecnica*, Dec. 1948, vol. 28, pp. 287–292.

After a brief introduction, the author discusses in conventional fashion the various conditions of vertical flight, those in which the momentum theory applies, and those in which it is no longer valid. He then writes the usual expressions for thrust and drag of blade elements, considered as airfoil elements. The elementary theory is applied to (1) vertical descent without power and (2) climb with motor. He also considers secondary terms and gives energy equations for the hovering condition. Numerical results are given that are of interest as showing the general direction of the theoretical approach. Alexander Klemin, USA

672. S. A. Sjoberg and J. P. Reeder, "Flight measurements of the longitudinal stability, stalling, and lift characteristics of an airplane having a 35° sweptback wing without slots and with 40-percent-span slots and a comparison with wind-tunnel data," *Nat. adv. Comm. Aero. tech. Note*, no. 1679, Aug. 1948, pp. 1–53.

This paper reports the results of flight tests, and a comparison with wind-tunnel tests, on a conventional fighter-type airplane with a sweptback wing of 35 deg at the quarter-chord line, an aspect ratio of 6.51 and a taper ratio of 0.544. Tests were also made with a slot that extended from the 40 per cent point to the 80 per cent point of the wing's semispan. The airplane was fully instrumented, and measurements of all the pertinent quantities and pictures of tufts on the wings are shown. Comparisons are made of flight test and wind-tunnel lift curves.

The longitudinal stability with the flaps up was high, with and without the slots. With the flaps down, the stability was high at moderate speeds but near the stall the stability was neutral or slightly negative. The airplane always pitched in a stable direction at the stall. Good stalling characteristics were obtained with the slots, but without them the stalling characteristics were poor.

It is very interesting to note that the slots reduced the maximum lift, and it is concluded that this is probably due to premature separation at the inboard end of the slot.

Conrad A. Lau, USA

673. Walter J. Klinar and Lawrence J. Gale, "Wind-tunnel investigation of the spinning characteristics of a model of a twin-tail low-wing personal-owner-type airplane with linked and unlinked rudder and aileron controls," *Nat. adv. Comm. Aero. tech. Note*, no. 1801, Jan. 1949, pp. 1–39.

The report presents the results of tests in the NACA's Langley 20-in. spin tunnel of an 1/11-size model. The model was tested as a two-control airplane with linked ailerons and rudders, and also with ailerons and rudders unlinked. Tests were made at two wing loadings and for mass distributions considered normal, primarily along the fuselage and along the wing.

As a two-control airplane, spins could not be induced with full-control wheel deflection right or left, but spins with partial wheel deflections could be induced. In any spin, recoveries were easily accomplished by reversing the controls.

On the basis of the model tests, complete spinproofing could be achieved by limiting the aileron to 5 deg up, by limiting the outboard rudder so that it would not be deflected with the spin, and by limiting the up-elevator deflection to 13 deg. Some modification to the size of the vertical tail appears necessary to provide satisfactory flying characteristics with the above control-surface deflection limitations. Conrad A. Lau, USA

674. **J. M. Evans and P. T. Fink, "Stability derivatives—determination of l_p by free rolling,"** *Austral. Counc. Aero. Rep.*, no. ACA-35, May 1947, pp. 1–23.

Continuing their experimental work on the lateral stability derivatives [see Revs. **1259** and **1262**, Aug. 1948], the authors describe a method of measuring the quantity l_p, the damping in roll. The free rotation of a model about the wind axis in a wind tunnel is used to determine the quantity l_p at angles of incidence below the stall. The rolling moment necessary to produce the rotation is obtained by a deflection of the ailerons, its magnitude being measured with the model stationary.

In the present case the rolling motion was disturbed by irregularities of the air stream. It is suggested that these irregularities are due to secondary flows arising in the corners of the nozzle of a wind tunnel with noncircular cross section. The effect of these disturbances could be eliminated to a large extent by the experimental technique and by a special calculation, so that the results of tests in two different tunnels were in good agreement.

The investigation was made on a $^1/_{10}$ scale model of the Boomerang single-engine fighter and on the wing alone. The measurements showed that at small lift coefficients the magnitude of the deflection of the ailerons was of no influence provided the ailerons were not stalled. But even when the ailerons were stalled the results were only slightly affected. The tests also confirm the known fact that in general the damping in roll l_p can be theoretically calculated with sufficient accuracy by means of the strip method.

Ernst Eujen, Germany

675. **Leonard Sternfield and Ordway B. Gates, Jr., "A simplified method for the determination and analysis of the neutral-lateral-oscillatory-stability boundary,"** *Nat. adv. Comm. Aero. tech. Note*, no. **1727**, Oct. 1948, pp. 1–45.

In this paper the authors call attention to recent investigations which have demonstrated that small variations in some of the airplane mass and aerodynamic parameters of several experimental high-speed airplanes may cause a pronounced change in the oscillatory stability of the airplane. The expression for neutral oscillatory stability involves Routh's discriminant for the lateral stability equation of a system having three degrees of freedom. This expression contains a large number of combinations of the mass and aerodynamic parameters. It therefore appears necessary to make a separate stability analysis for each airplane.

The theoretical analysis presented in this note demonstrates that Routh's discriminant, in general, takes the form $R = D(BC - AD) - B^2E$, where A, B, C, D, and E are the coefficients of the lateral-stability equation. On a large number of cases considered, it has been found that the term B^2E may be neglected. Routh's discriminant is then factorable in the form $R = D(BC - AD)$. In such cases, either $(BC - AD) = 0$, or $D = 0$ may constitute a condition of neutral stability. If these conditions for a neutral stability boundary are satisfied, then $D = 0$ represents the boundary for an oscillation which has a comparatively larger period than the period of the oscillation for which $(BC - AD) = 0$ is the boundary.

The comparative simplicity of the two factors of R provides an insight into the important combinations of mass and aerodynamic parameters which affect the lateral oscillatory stability. It is suggested by the authors that this insight into the role played by these major parameters on oscillatory stability might suggest methods of analysis which would eliminate the need for separate calculations for each airplane.

The note is concluded with a discussion of the nature of the modes of motion as a function of the directional stability derivative and the effective dihedral angle. Calculations based on the simplified expressions are presented for comparison with the results obtained by the use of the complete expression for the neutral oscillatory stability boundary. Louis A. Pipes, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Rev. 680)

676. **S. I. Pai and W. R. Sears, "Some aeroelastic properties of swept wings,"** *J. aero. Sci.*, Feb. 1949, vol. 16, pp. 105–115.

The problem of finding the effect of elasticity on the lift distribution of sweptback and sweptforward wings is examined. The study is restricted to wings that have one dimension, normally the span, much greater than the chord, so that the basic assumptions of Prandtl's lifting line wing theory can be applied; however, methods of refining the Prandtl theory for highly swept wings are also indicated. The solution is presented in the form of an integral equation for the local lift coefficient. This equation is inverted by approximate numerical methods and the results are presented in matrix notation.

Sample calculations are presented for the span lift-distribution, the torsional divergence, the aileron effectiveness and reversal speed, and the dive pull-out. The known aeroelastic effects on sweptback and sweptforward wings are confirmed, and it is shown that in some cases these effects can be large. A case involving an airplane oscillation with wing bending is also investigated, although the mode of wing bending deflection is assumed. The results for this case show reduced static stability and damping in pitch for the elastic wing. Harvard Lomax, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 569, 570, 671, 684, 685)

677. **Daniel Gaden and Pierre Neeser, "On the subject of automatic control of the speed of hydraulic turbines—influence of certain characteristics on the stability condition (À propos du réglage automatique de vitesse des turbines hydrauliques—Influence de certaines caractéristiques intervenant dans la condition de stabilité),"** *Houille blanche*, May-June 1948, vol. 3, pp. 256–271.

This paper discusses the stability of the automatic speed control of hydraulic turbines studied by means of two methods of taking into account the effect of the water hammer: (1) The water is assumed to be exactly incompressible and the conduits to be absolutely rigid. (2) The slight compressibility of the water and the deformability of the walls of the conduits are not neglected, the effect of water hammer being considered as a wave phenomenon.

By means of the first method a basic equation is developed as a condition of stability:

$$\tau' \cdot T > K\,(3\Theta/2)^2$$

where τ' is a time characterizing the rapidity of response of the governor system, T a time characterizing the inertia of the revolving masses, and Θ a time characterizing the inertia of the fluid masses filling the conduits.

In this equation K is a dimensionless constant provided the first assumption is exact enough, and some other assumptions describing the behavior of the turbine-governor system are valid.

Next, applying the second method and reconsidering the assumptions just mentioned, the paper gives a thorough discussion of the problem, as a result of which K turns out to be not a constant but a function of a number of characteristics the influence of which is shown in diagrams.

To the reviewer the paper seems to be of considerable practical value. Wilhelm Spannhake, USA

678. E. R. Love and J. P. O. Silberstein, "Vibration of stationary and rotating propellers," *Austral. Counc. Aero. Rep.*, no. ACA-36, June 1947, pp. 1–54.

The paper starts with a detailed review of the literature on propeller vibration. This review unfortunately omits the important early paper of Hans Reissner [*Ingen.-Arch.*, 1933, vol. 4, pp. 557–569] which gives the theory for the torsional as well as the flexural vibration of a rotating propeller blade with initial twist and curvature. The authors develop first the equations for the flexural vibration of a nonrotating propeller blade. The blade is assumed slender and of gradually varying section, with centroids lying on a line at right angles to the axis of the propeller shaft when at rest. The root of each blade is rigidly fastened to a hub on the propeller shaft.

The normal vibration in the plane of each section of the blade (normal to the x-axis) is expressed as $v(x, t) = V(x) \sin pt$, $w(x, t) = W(x) \sin pt$ and the rotation about the propeller axis at the propeller root by $\Theta(t) = \Theta \sin pt$. Three integral equations are derived for the curvature amplitudes $V''(x)$, $W''(x)$ and the amplitude of rotational vibration Θ, from the known dimensional constants (moments of inertia, cross-sectional area, etc.) of the blade, and from the Young's modulus and density of the blade material The equations involve single rather than multiple integrals, thanks to the use of curvature amplitude rather than displacement amplitude and thanks to the introduction of a function $a(x, \xi)$ which depends only on the distribution of cross-sectional area along the blade. The solution is subsequently extended to cover the case of a propeller which vibrates while it is driven by a torsionally elastic shaft.

It is shown how the three integral equations for V'', W'', Θ may be solved approximately by dividing the range of integration into n equal intervals, representing the integrands by their values at the mid-points of the intervals, and replacing the integrals by sums. This leads to $2n + 1$ homogeneous linear equations in terms of the $2n + 1$ unknowns $V''(x_i)$, $W''(x_i)$, Θ. The determinant of the coefficients of these equations may be used to compute the relation between torsional impedance and frequency of the propeller. The procedure is illustrated numerically by applying it to a de Havilland three-blade propeller.

It is shown in an appendix that the lateral bending of the blades due to aerodynamic loading has a negligible effect on frequency in the case of propellers of conventional design.

Walter Ramberg, USA

679. C. D. Denney, "Design criteria for jet-propelled helicopter rotors," *Trans. Amer. Soc. mech. Engrs.*, Jan. 1949, vol. 71, pp. 1–8.

The author points out that a great interest exists in the application of jet propulsion to helicopter rotors. Efforts have been made to construct a complete aircraft on this principle and to determine its flight characteristics. From among the many different types of jet drive the author considers two only. One, of extreme simplicity, is known as the ramjet system, in which a ramjet engine is mounted on each rotor tip. The second system is the pressure system in which an internal-combustion engine drives an air compressor in the fuselage, while the pressurized air is driven through the hub to a constant-pressure burner mounted at the blade tip.

The paper considers in detail the performance of the ramjet system and of the pressure system under different conditions. The most important part of the paper is in the comparison of performance. The ramjet propulsion system consumes a great deal more fuel per hour than an internal-combustion engine driving the rotor directly and producing equivalent power. The pressure system is also more expensive in fuel than the combustion engine, but to a smaller degree than the ramjet. The advantage of either system over a conventional rotor drive lies in a reduction of weight by eliminating the auxiliary torque-compensating rotor, the clutch, free wheeling and transmission assemblies and the large internal-combustion engine.

For a ramjet helicopter, a greater pay load can be carried up to an endurance of $1^1/_4$ hr. For an aircraft of 4800 lb, if the flight is only $^1/_2$ hr, as much as 1000 lb additional pay load can be carried by ramjet helicopters. For an equivalent helicopter using the pressure-type jet-propulsion system, a greater pay load can be carried up to an endurance of $2^1/_4$ hr. These characteristics of jet-propelled helicopters should make them suitable for specialized applications where short endurance only is required.

Alexander Klemin, USA

680. S. W. Yuan and M. Morduchow, "On the stability of the transient motion of helicopter blades in flapping and lagging," *Reissner Anniv. Vol.*, J. W. Edwards, Ann Arbor, 1949, pp. 163–181.

The present paper extends a method introduced in a paper by G. Horvay and S. W. Yuan [see Rev. **397,** Mar. 1948]. In the present work an additional degree of freedom is permitted in the blade motion, namely, the blade lagging about a vertical hinge. It is shown that the coupling between flapping and lagging is weak and consequently the results differ little from the previous work. It is shown that the motion of the blade is stable except for large negative angles of δ_3, specifically $\delta_3 = -35$ deg.

Arthur A. Regier, USA

681. John Katzman, "The determination of the efficiency of a hydraulic turbine by a calorimetric method," *Canad. J. Res. Sec. F*, Dec. 1948, vol. 26, pp. 513–522.

Special thermometers were used to determine the difference in temperature of the water between the headrace and tailrace of a hydraulic turbine at different loads on the turbine. From this temperature difference the efficiency of the turbine was determined and the volume rate of water flow was calculated.

R. C. Binder, USA

Flow and Flight Test Techniques

(See also Revs. 659, 681)

682. W. Hausammann, W. Gaehler, G. Kogen, J. Vetterli, R. Diem, B. Graemiger, "The windtunnel plant of the Swiss Bureau of Ordnance in Emmen (Die Windkanalanlage der Kriegstechnischen Abteilung in Emmen)," *Schweiz. Bauztg.*, 1948, vol. 66: Sept. 25, pp. 527–535; Oct. 9, pp. 563–571; Nov. 13, pp. 627–634; Nov. 20, pp. 644–648.

An aeronautical laboratory for the development testing of aircraft under construction by the Swiss Federal Aircraft Establishment is described. The completed phase of the installations con-

tains two low-Mach-number wind tunnels and a spinning tower. The larger of the tunnels (23 × 16-ft working section, 270 ft per sec air speed) can be operated with open or closed throat; it is equipped with air exchangers, honeycomb, a two-stage counter-rotating fan of 28-ft diam with a 3900-hp drive. The articles contain a well-illustrated description of the buildings, the tunnel construction, electrical equipment and balance system with automatic recording devices. The small tunnel has a working section of 8 × 6 ft and is combined with the vertical spinning tower; the change-over to the spinning configuration is accomplished by a rotation of one of the tunnel corner sections by 90 deg. Data are also presented on flow distribution in the test section, the stability of the balance system, the accuracy of the weighing elements and calibration.

Andrew Fejer, USA

683. **Geneviève Dubois and Roger Kling, "On the measurement of fast gas current characteristics by ultrasonic means (Sur la mesure des caractéristiques d'un courant gazeux rapide au moyen d'un faisceau d'ultrasons),"** *C. R. Acad. Sci. Paris*, Jan. 31, 1949, vol. 228, pp. 363–364.

The speed of an air stream in a wind tunnel is measured by the deflection of a beam of ultrasonic waves entering an orifice on one side of the tunnel and reflected by the opposite wall.

W. G. Bickley, England

Thermodynamics

(See also Revs. 662, 665, 685)

684. **R. F. Davis, "Steam and air turbine generating plants,"** *Engineering*, Jan. 21, 1949, vol. 167, pp. 49–50.

Two types of combined steam and air-turbine plants for large-capacity power generation are discussed. One plant for power generation only (no process steam requirements) uses a steam-driven condensate intercooled compressor. The load is carried by a reheated air turbine, the exhaust of which is fed through a regenerator and thence to a feed-water heater. The use of an externally fired heater enables solid fuels to be used. The thermal efficiency without auxiliaries is estimated to be 37 per cent.

The second plant for generation of power and process steam uses two generators. One is driven by the conventional air turbine and compressor combination, the other by a steam turbine. The estimated efficiency of this plant is 52 per cent.

C. A. Meyer, USA

Heat Transfer; Diffusion

(See also Rev. 689)

685. **A. L. London and C. K. Ferguson, "Test results of high-performance heat-exchanger surfaces used in aircraft intercoolers and their significance for gas-turbine regenerator design,"** *Trans. Amer. Soc. mech. Engrs.*, Jan. 1949, vol. 71, pp. 17–26.

This paper presents a summary of experimental tests on steam-to-air exchangers employing various types of extended surfaces in the form of louvered fins. Friction factors and heat-transfer coefficients were then calculated by standard methods from these data. The different kinds of extended surfaces were then compared with each other and with the case of flow through round tubes of the same hydraulic diameter on the basis of a plot of heat-transfer coefficient versus friction horsepower, analogous to plots used previously by Colburn. The results obtained are analyzed in terms of the laminar and turbulent boundary layers. The authors conclude in part that data for flat plates can be used to predict the heat-transfer characteristic for plate-fin surfaces within 30 per cent provided the type of Prandtl boundary layer can be specified.

The data obtained are also investigated from the point of view of application in the design of regenerators for gas-turbine power plants. Here the primary considerations are given as manufacturing limitations, cost, cleaning, weight, space, shape, etc. A comparison of turbulent flow inside round tubes with flow over flat plates, and with turbulent flow normal to a bank of in-line cylinders leads to the conclusions, in part, that "flow inside tubes has a poor heat transfer-friction power characteristic," "the flat-plate flow with a laminar boundary layer has a better characteristic than the flow with an eddy-type boundary layer," etc. The substantiation of these points does not seem conclusive.

Joseph Kaye, USA

Acoustics

(See also Rev. 572)

686. **R. L. Abbey and G. E. Barlow, "The velocity of sound in gases,"** *Austral. J. sci. Res. Ser. A*, June 1948, vol. 1, no. 2, pp. 175–187.

The authors have made velocity of sound measurements on air, nitrogen, oxygen, carbon dioxide, and methane, using an acoustic feed-back method with the generator and detector of sound placed at the ends of a closed tube. The measurements covered a range of pressures from atmospheric down to a few millimeters of mercury. Tube effects were corrected by the classical Helmholtz-Kirchoff formulas

$$V_r = V[1 - c/(2r\pi^{1/2}n^{1/2})]$$

and

$$c = (\eta/\rho)^{1/2} + [K/(\rho C_v\gamma)]^{1/2}(\gamma - 1)$$

where V_r is the observed velocity in a tube of radius r at a frequency n, while η, ρ, K, γ and C_v are, respectively, the viscosity, the density, the conductivity, the ratio of specific heats, and the specific heat at constant volume of the gas.

The authors point out that the correction term c is inversely proportional to the square root of the density of the gas and thus is of increasing importance at low pressures, a point often overlooked by earlier investigations.

The reported corrected measurements are in good agreement with the work of others at atmospheric pressures. There seems to be an increase in velocity as the pressure is reduced. Methane showed an anomalous drop near 1.5 mm of Hg followed by a rise. This change of velocity is concluded to be either real or due to a failure of the Helmholtz-Kirchoff equations at low pressures. The subject is to be investigated further.

Horace M. Trent, USA

Ballistics, Detonics (Explosions)

(See also Revs. 565, 634)

687. **T. Felsztyn, "Motion of a finned projectile on its trajectory (Le mouvement du projectile empenné sur sa trajectoire),"** *Mémor. Artill. fr.*, 1948, vol. 22, no. 2, pp. 485–514.

This paper considers the vertical yawing motion of a fin-stabilized projectile (in particular, a mortar shell), and its effect on range and dispersion. Besides gravity and drag, the projectile is subject to aerodynamic restoring and damping moments, of the form $(\beta\theta + \gamma\theta^3)v^2$ and $Krd\theta/dt$ respectively, where v is the velocity and θ is the angle of yaw. Only low velocities are considered. Illustrative computations are given and there is a long discussion of the stability of mortar shells at high angles of fire.

A. S. Galbraith, USA

Soil Mechanics, Seepage

688. F. K. Th. van Iterson, "Earth pressure in mining" (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 3, pp. 314–316.

Points of similarity are established between problems of soil mechanics and certain problems in mining. It is shown that at the great depths at which coal mining is performed in Holland the rock which adjoins the timbering of a tunnel is bound to fracture and to be transformed into loose material to which the laws of equilibrium of granular soils (with internal friction only governing the shearing strength) can be applied.

Good agreement is claimed between the angle of break of the rock above abandoned areas from which coal had been removed, as computed in accordance with the above laws and as determined from the observed subsidence of the ground surface.

It is shown that at some distance from a cylindrical tunnel the radial pressure in the rock reaches a maximum value equal to 1.7 times the overburden pressure. Agreement is claimed between the above computed value and actual pressure measurements in English mines. It is further shown that the vertical pressure on the coal front of "longwall workings" can be determined by applying Prandtl's solution to the problem, but in a manner which reverses the procedure used in soil mechanics when the effects of a load on the neighboring ground surface are studied.

Gregory P. Tschebotarioff, USA

Geophysics, Meteorology, Oceanography

(See also Rev. 691)

689. R. B. Montgomery, "Vertical eddy flux of heat in the atmosphere," *J. Met.*, Dec. 1948, vol. 5, pp. 265–274.

The first part of the paper is devoted to the thermodynamics of a special atmospheric system regarded as an open system (a system which can exchange matter through the upper surface). The upward-eddy flux of heat per unit area is given by $\overline{\rho V_z' h}$ where ρ is the density, V_z' a specially defined fluctuation of the vertical component of velocity and h the specific enthalpy. This expression leads to special ones with the flux of latent heat both excluded and included, and to expressions for the eddy flux of heat from dry ground and from a large water surface. All these expressions are derived from the fundamental equation:

$$\overline{\rho V_z' h} = -K\overline{\rho}\partial\overline{Y}/\partial z$$

where K is the coefficient of eddy diffusion and Y is the sum of specific enthalpy h plus geopotential φ.

J. Kampé de Fériet, France

690. H. C. Willett, "Long-period fluctuation of the general circulation of the atmosphere," *J. Met.*, Feb. 1949, vol. 6, pp. 34–50.

This paper reviews facts established by previous investigators and the theories advanced to explain them. The essential similarity of the various climatic changes which have occurred during observational time, during historical time, and during the geological past is stressed, and it is pointed out that the only single factor which could account for them all is solar variability of the sunspot variety.

M. V. Wilkes, England

Lubrication; Bearings; Wear

(See Rev. 656)

Marine Engineering Problems

(See also Revs. 569, 570)

691. H. Maruo, "A contribution to the theory of a gliding plate on the surface of water" (in Japanese), *J. Soc. nav. Archit. Japan*, Oct. 1947, no. 78, pp. 1–15.

If a two-dimensional pressure disturbance $p(x)$ $(-l \leqq x \leqq l)$ advances with constant velocity V over the water surface, the profile of the pressure surface and the resistance experienced by it are given by

$$y(x) = \frac{1}{\pi\rho V^2} \int_{-l}^{l} p(\xi)d\xi \int_0^{\infty} \frac{\cos\alpha(x-\xi)}{\alpha - g/V^2}\, d\alpha,$$

$$R = -\int_{-l}^{l} p(x)\frac{dy}{dx}\,dx$$

respectively [H. Lamb, *Hydrodynamics*, 6th ed., p. 399]. When $p(x)$ is finite, R is found to be proportional to the gravity acceleration g; it represents the wave resistance. The author shows that the pressure distribution $p(x) = [(l-x)/(l+x)]^{1/2}$ implies, in addition to the wave resistance, a resistance independent of g, and that the new resistance results from the splash formation at the leading edge $x = -l$, where $p(x)$ becomes infinite. The solution for a given gliding plate may be obtained by assuming the pressure distribution in the form

$$a_0[(l-x)/(l+x)]^{1/2} + (a_1 + a_2 x)(l^2 - x^2)^{1/2},$$

and determining the coefficients so that $y(x)$ coincides with the given profile.

The author's analysis gives the total resistance, in which the wave resistance is predominant at a small Froude number $V/(2gl)^{1/2}$ while the splash resistance is predominant at a large Froude number. In the limit of an infinite Froude number, the resistance is purely due to splash formation, and the solution gives a result which agrees approximately with Green's exact theory for the case of no gravity. The effect of profile form on the splash resistance is considerable, the camber especially giving a favorable effect. The extension of the solution to the three-dimensional case is also suggested.

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